RESEARCH ARTICLE

AN APPLICATION OF MULTIVARIATE STATISTICAL PROCESS CONTROL IN DANA STEEL COMPANY

*Ibrahim Mu’awiyya Idris

Umaru Musa Yar’adua University Katsina, Nigeria

ARTICLE INFO

Article History:
Received 15th June, 2014
Received in revised form 06th July, 2014
Accepted 10th August, 2014
Published online 18th September, 2014

Key words:
Multivariate Statistical Process Control, Hotelling’s T-square statistic.

ABSTRACT

Multivariate statistical process control is a branch of industrial statistics that involve monitoring quality specification of related variables simultaneously. Quality is the most essential target for manufacturing engineers and which mostly involves more than one variable in industry, i.e., a vector of variables (that conform to specification for measurement) which may be correlated. When these quality variables are correlated then the most well-known approach for multivariate process monitoring is the Hotelling’s T-square control chart. In this research, a multivariate data in subgroups consisting of five quality characteristics obtained from Dana Steel Company Limited Katsina is analyzed for quality. Retrospective analysis shows that, the production process from which the data were obtained is in statistical control.

INTRODUCTION

Multivariate statistical process control is a branch of industrial statistics that deals with two or more related quality variables (quality characteristics), the first pioneering work in the study of multivariate quality control was done in Hotelling (1931). Hotelling wrote a paper on T-square test procedures for multivariate population and subsequently in Hotelling (1947), hotelling developed an extension of the T-square to control charts. Often there exist variability in the raw materials and variability in the production process due to natural and assignable causes, in addition the effect of these variabilities directly affects the products conformed to quality specification. The growing interest in multivariate quality control started in the last three decades when computer technology record its advancement that makes it possible to monitor multiples of quality variable all together in production process. An extensive review of literature for multivariate quality control (MQC) which includes multivariate cumulative sum (MCUSUM) and Multivariate exponentially weighted moving average (MEWMA) are discussed in Lowry and Montgomery (1995) and also in Alt (1985). In Timothy and Paul (1999), the multivariate control chart of Hotelling T-square was applied to a wood industry to monitor the vertical density profile of the wood which is influenced by many quality variables in the manufacturing process, there are indeed many application of Hotelling T-square multivariate control chart in literature.

This research work focuses on the application of Hotelling T-square multivariate control chart in Dana steel rolling industry Katsina state, Nigeria. Recently there were sudden collapses of buildings across the nation and which claimed lives and properties worth millions of naira, research and investigations revealed that many factors contributed to the disaster and one of these factors is whether or not steel rods used during construction processes met the quality requirements. The data used in this research is a secondary data in ten subgroups of ten observations each obtained from Dana steel company limited Katsina, it comprises of five quality variables (Weight KG/M, Area MMSQ, Breaking force KGF/MMSQ, Tensile strength N/MMSQ and Yield strength N/MMSQ).

METHODS

Hotelling T-square control chart for sub-grouped data is used, the test statistics is given as

\[ T_i^2 = n(\bar{X}_i - \bar{X})' S_p^{-1}(\bar{X}_i - \bar{X}) \]  

Where \( \bar{X}_i \) is the sample mean of size \( n \) for the \( m \) subgroups for which \( i = 1, 2, ..., p \), \( \bar{X} \) is an unbiased estimator for the unknown population mean and \( S_p^{-1} \) is the inverse of the pooled covariance matrix which is obtained by averaging the subgroup covariance matrices over the \( k \) subgroups. There two phases for control chart usage, the control limit for phase (I) known as the retrospective analysis is used for monitoring the in-control condition for the process under investigation.
Phase (II) which is known as the prospective analysis is used to set a control limit for future production having identified the quality attribute responsible for the out of control signal, in addition, this stage is known as the stage of improving quality, Ryan (1989). The control limit for this phase is given as:

\[
UCL = \frac{p(m-1)(n-1)}{mn - m - p + 1} F_{a(p, mn - m - p + 1)} \\
LCL = 0
\]

Analysis of the sub grouped data (retrospective analysis)

The five \( p \) quality variables are jointly monitored from the sample data consisting of \( m \) subgroups with \( n \) observations each, the analysis is presented in the following steps

**Step1:** Compute the sample mean \( \bar{X} \) for each of the 10 subgroups

Subg1 \( \bar{X} = [0.8542 \ 108.7000 \ 7808.0000 \ 719.2000 \ 451.8000] \)
Subg2 \( \bar{X} = [0.8661 \ 110.4000 \ 7190.0000 \ 650.9000 \ 430.6000] \)
Subg3 \( \bar{X} = [0.8503 \ 108.3000 \ 7172.5000 \ 662.6000 \ 430.9000] \)
Subg4 \( \bar{X} = [0.8528 \ 108.5000 \ 7360.0000 \ 678.2000 \ 431.0000] \)
Subg5 \( \bar{X} = [0.8561 \ 109.0000 \ 7283.0000 \ 669.7000 \ 435.7000] \)
Subg6 \( \bar{X} = [0.8689 \ 109.6000 \ 7572.5000 \ 680.7000 \ 442.1000] \)
Subg7 \( \bar{X} = [0.8585 \ 109.3000 \ 7872.5000 \ 719.6000 \ 458.9000] \)
Subg8 \( \bar{X} = [0.8631 \ 110.0000 \ 7561.0000 \ 687.8000 \ 439.1000] \)
Subg9 \( \bar{X} = [0.8792 \ 112.0000 \ 7265.0000 \ 648.6000 \ 422.3000] \)
Subg10 \( \bar{X} = [0.8666 \ 110.3000 \ 7464.0000 \ 679.0000 \ 428.2000] \)

**Step2:** Compute the sample mean average \( \bar{X} \) (Grand mean) of the subgroups by averaging over the sub grouped means.

\[
\bar{X} = \frac{\bar{X}_1 + \bar{X}_2 + \ldots + \bar{X}_n}{n}
\]

\( \bar{X} = (0.86158 \ 109.71000 \ 7453.35000 \ 679.56000 \ 437.06000) \)

**Step3:** Set the matrices \( (\bar{X} - \bar{X}) \) and transpose the matrices \( (\bar{X} - \bar{X})' \).

Subg1

\[
(\bar{X} - \bar{X}) = \begin{bmatrix}
-0.01158 \\
-0.71000 \\
2196.65000 \\
214.44000 \\
-6.06000
\end{bmatrix}
\]

This is computed for the remaining subgroups. Summing up the matrices \( (\bar{X} - \bar{X}) \) and \( (\bar{X} - \bar{X})' \) we obtain the following matrices

\[
(\bar{X} - \bar{X}) = \begin{bmatrix}
1.435967 & 182.850000 & 12422.250000 & 1132.600000 & 728.433333
\end{bmatrix}
\]

**Step4:** Compute the sample variance-covariance matrix \( S \) for each of the subgroups. For subgroup 1, the covariance is computed as

\[
\frac{0.0001081778 \ 0.011955556 \ -8.4684444 + 0.00 -8.6162222e01 -9.155556 + 03}{0.019555556 \ 1.34444444 + 0.02 -1.0071111e02 \ 2.488889 + 00}
\]

The covariance matrices for the remaining \( n - 1 \) subgroups are computed respectively.

**Step5:** Set the pooled covariance matrix \( S_{pooled} \) by averaging the covariance matrices of the nsubgroups and compute the inverse pooled covariance matrix \( S^{-1}_{pooled} \) which is given as

\[
\begin{bmatrix}
488.788637 & 3.8657026274 & 202171e + 00 - 6.363880e00 - 01 - 2.280665e + 00 \\
3.8657026274 & 0.029972525 & 2.706098e - 02 - 4.895473e - 03 - 1.578866e - 02 \\
2.02171e + 00 & 0.0270608807 & 3.088812e + 07 - 3.631240e - 00 - 7.143994e - 05 \\
6.363880e00 - 0.0049754733 & 3.631240e - 00 & 3.456307e + 00 - 6.296908e - 04 \\
-2.280665e + 00 - 0.01758627143944e - 05 & 6.296908e - 04 & 3.904907e - 04
\end{bmatrix}
\]
Step 7: Compute the Hotelling T-square statistic using eqn (1) which is given by

\[
\begin{bmatrix}
106043677 & 40133022 & 38688899 & 55294991 & 48106872 & 77674311 \\
113560407 & 76361994 & 46567964 & 65800985 & 113560407 & 76361994 \\
\end{bmatrix}
\]

Step 8: Using eqn (2) and choosing \( \alpha = 0.01 \), we compute the upper control limit where

\[
UCL = 15.72 \\
LCL = 0
\]

The corresponding control chart for the retrospective analysis is given in Fig 1, where \( T^2 = Y_i \) values are plotted against the subgroups \( X_i \).

RESULTS AND DISCUSSION

In our analysis of the sub-grouped data, all the subgroups points fall within the control region which are below the upper control limit as indicated by the control chart (Fig. 1), therefore the production process that generated the data is said to be in state of statistical control. In the analysis of Hotelling T-square for sub-grouped data, since there are two distinct phases for control chart usage where phase I (retrospective analysis) is used in testing whether the process is in state of statistical control when \( m \) initial samples were drawn and the sample statistic \( \bar{X} \) and \( S_{\text{pooled}} \) computed for the subgroups. In addition, the main objective is to obtain an in control set of observations so that control limit can be established for phase II, but if in examining the control chart of phase I (retrospective examination) and all points plot within the control limit as in Figure 1, i.e. no point exceed \( UCL \), then the same control limit \( UCL \) for phase I is sufficient for monitoring the future process of production, hence, \( UCL \) for phase II will no longer be computed.

Conclusion

Quality is the most essential factor within market, and multivariate control chart procedure using Hotelling’s T-square statistic provides a reasonable tool for monitoring production processes with multiple correlated variables. In this research we have analyzed a multivariate data obtained from Dana Steel Company Limited Katsina, Nigeria and the analysis carried out based on Hotelling’s T-square revealed that the data was obtained from statistically controlled production process.

REFERENCES


********