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RESEARCH ARTICLE

TABULAR INTEGRATION BY PARTS THE BEST SHORT CUT TO PERFORM INTEGRATION

*Salwa Mrayyan

Al-Balqaa Applied University, Jordan

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ABSTRACT

This suggested method is applicable to all problems that can be integrated by Engineering students are required to know too much math, they also need to master methods of computing integrations analytically, i.e., integrating by parts. Integrating by parts using the (shortcut) or tabular integration makes integration clear, neat, and accurate. The method has been known for a long time; however no one seems to give it its true value. In this research, the researcher introduced the method after doing the needed modifications so it may be applicable for all math problems which can be integrated by parts. From the experience of teaching calculus and other advanced math courses, the researcher found out that student who used (TIBP)method were more accurate and faster in the exams if compared with those who used the traditional method, Integration by parts is important to all scientists and engineers as well as to mathematicians. Comparing integration by parts using the traditional method is considered to be long, misleading, and sometimes hard for average and good students, especially if it had negative signs and fractions. The tabular integration technique is suggested as an alternative method to ease solving problems and to allow one to perform successive integration by parts on integrals of the parts, and it also can be used to prove some theorems such as Taylor Formula, Residue Theorem for Meromorphic Functions and, Laplace Transformation theorem, as well as evaluating the integral of the product of three functions This method is fast, feasible, and applicable, it strengthens students' confidence in their work.

INTRODUCTION

Engineering students are required to know too much math, they also need to master methods of computing integrations analytically, i.e., integrating by parts. Integrating by parts using the (shortcut) or tabular integration makes integration clear, neat, and accurate. The method has been known for a long time; however no one seems to give it its true value. In this research, the researcher introduced the method after doing the needed modifications so it may be applicable for all math problems which can be integrated by parts. From the experience of teaching calculus and other advanced math courses, the researcher found out that student who used (TIBP) method were more accurate and faster in the exams if compared with those who used the traditional method, Integration by parts is important to all scientists and engineers as well as to mathematicians. Comparing integration by parts using the traditional method is considered to be long, misleading, and sometimes hard for average and good students, especially if it had negative signs and fractions. The tabular integration technique is suggested as an alternative method to ease solving problems and to allow one to perform successive integration by parts on integrals of the form $\int f(t)g(t)dt$.

This suggested method is applicable to all problems that can be integrated by parts, and it also can be used to prove some theorems such as Taylor Formula ,Residue Theorem for Meromorphic Functions and, Laplace Transformation theorem, as well as evaluating the integral of the product of three functions This method is fast, feasible, and applicable, it strengthens students' confidence in their work.

Literature Review

To the best of the researcher's knowledge, no similar published researches dealt with the same issue, the only research that dealt with this method is Horowit who wrote: "Only a few contemporary calculus textbooks provide even a cursory presentation of tabular integration by parts ,This is unfortunate because tabular integration by parts is not only a valuable tool for finding integrals but can also be applied to more advanced topics including the derivations of some important theorems in analysis. "This technique of tabular integration, as Horowit argues, allows one to perform integrals of the form $\int f.g\,dx$ without becoming bogged down in tedious mathematical details" (Horowit, 1990). While V. N. Murty, found that there are several ways to illustrate this method, one of which is diagrammed in tables for differentiations and integrations, assuming throughout that *F* and *G* are "smooth" enough to allow repeated differentiation and integration, respectively, (Murty, 1980. 90-94). According to Jaime Escalante the technique of tabular integration by parts makes an appearance in the hit motion picture *Stand and Deliver* in which mathematics instructor works the following example.

$\int x^2 \sin x \, dx$

$+x^2$ $\sin x$

$-2x$ $-\cos x$

$+2$ $-\sin x$

0 $+\cos x$

The answer: $-x^2 \cos x + 2x \sin x + 2\cos x + C$

As Horowitz shows, the method of TIBP is a blockbuster, as can be seen from the way it knocks off an integral like, $\int \frac{12t^2+36}{\sqrt[5]{3t+2}} dt$. On the other hand McDowell used the method to introduce the following example $\int \sec^3 x \, dx$, he didn't develop it, while the researcher developed it and added a solution of the integration of product of three functions as shown below From what is mentioned above, researchers seem to explain the methods in, different stages ,examples without studying the effectiveness of the method on the students' achievements.

Tabular integration by parts method (TIBP)

The tabular integration technique allows one to perform successive integration by parts on the integral of the form $\int f(t)g(t)dt$

It is almost applicable to all problems that can be integrated by parts, it can be used to prove some theorems such as Taylor Formula, Residue Theorem for Meromorphic Functions and, Laplace Transformation theorem, and can be used to evaluate the integral of the product of three functions as well.

Case 1 (product of polynomials and exponential functions)

$\int p_n(x).e^{mx}$, where $p_n(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$

Step1 Is the Set up of the two function in two columns

For differentiation	For integration
$p_n(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$g(x) = e^{mx}$

One for differentiation and the other for integration differentiation respectively As an example let us take a polynomial of degree 3, $p_3(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ and the function $g(x) = e^{mx}$, the aim is to find the integration of $p_3(x) \cdot e^{mx}$

Step 2

Start the process of integration and differentiation (keep differentiation until we got 0)

For differentiation	For integration
$a_3x^3 + a_2x^2 + a_1x + a_0$	e^{mx}
$3a_3x^2 + 2a_2x + a_1$	$\frac{e^{mx}}{m}$
$6a_3x + 2a_2$	$\frac{e^{mx}}{m^2}$
$6a_3$	$\frac{e^{mx}}{m^3}$
0	$\frac{e^{mx}}{m^4}$

Step 3 To sign up

signs	For differentiation	For integration
+	$a_3x^3 + a_2x^2 + a_1x + a_0$	e^{mx}
-	$3a_3x^2 + 2a_2x \ + a_1$	$\frac{e^{mx}}{m}$
+	$6a_3x \ + 2a_2$	$\frac{e^{mx}}{m^2}$
-	$6a_3$	$\frac{e^{mx}}{m^3}$
+	0	$\frac{e^{mx}}{m^4}$

Step 4 Matching

signs	For differentiation	For integration
+	$a_3x^3 + a_2x^2 + a_1x + a_0$ ↘	e^{mx}
-	$3a_3x^2 + 2a_2x \ + a_1$ ↘	$\frac{e^{mx}}{m}$
+	$6a_3x \ + 2a_2$ ↘	$\frac{e^{mx}}{m^2}$
-	$6a_3$ ↘	$\frac{e^{mx}}{m^3}$
+	0 →	$\frac{e^{mx}}{m^4}$

Step 5 Compute the final answer, which will be the product of matched items the answer will be

$$(a_3x^3 + a_2x^2 + a_1x + a_0)\left(\frac{e^{mx}}{m}\right) - (3a_3x^2 + 2a_2x \ + a_1)\frac{e^{mx}}{m^2} + (6a_3x \ + 2a_2)\left(\frac{e^{mx}}{m^3}\right) - (6a_3)\frac{e^{mx}}{m^4} + C$$

Case 2 (The product of exponential functions and trigonometric functions in general)

$$\int e^{ax} \sin bx dx$$

The following table shows the problem setup

signs	For differentiations	For integration
+	e^{ax}	$\sin bx$
–	ae^{ax} ↘	$-\frac{\cos bx}{b}$
+	a^2e^{ax} ↘	$-\frac{\sin bx}{b^2}$
	$+ \int$ →	

The final work for the problem will be,

$$\int e^{ax}\sin bx\,dx = -\ e^{ax}\frac{\cos bx}{b} + ae^{ax}\frac{\sin bx}{b^2} - \int a^2e^{ax}\frac{\sin bx}{b^2}\,dx$$

If you look at the last term you will notice that it is the same problem we started with except little constants change , so if we move the last term before the equal sign we will get ,

$$\int e^{ax}\sin bx\,dx + \int a^2e^{ax}\frac{\sin bx}{b^2}\,dx = -\ e^{ax}\frac{\cos bx}{b} + ae^{ax}\frac{\sin bx}{b^2}$$

$$(1+\frac{a^2}{b^2})\int e^{ax}\sin bx\,dx = -.\ e^{ax}\frac{\cos bx}{b} + ae^{ax}\frac{\sin bx}{b^2}$$

And $\int e^{ax}\sin bx\,dx - = (\frac{b^2}{a^2+b^2})\ (-\ e^{ax}\frac{\cos bx}{b} + ae^{ax}\frac{\sin bx}{b^2})+C$

Case 3 Product of polynomial and logarithmic functions

Let us consider the problem $\int p_n(x)\ln x\,dx$

where $p_n(x) - = -a_nx^n + a_{n-1}x^{n-1} + \cdots + a_0$,

signs	For differentiation	For integration
+	$\ln x$	$a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0$
−	$\frac{1}{x}$	$\frac{a_n}{n+1}x^{n+1} + \frac{a_{n-1}}{n}x^n + \ldots + a_0x$
	$+\int$	

From the table the integral will be:

$$\int (a_nx^n + a_{n-1}x^{n-1} + \ldots + a_0)\ln x\,dx =$$

$$\ln x\ (\frac{a_n}{n+1}x^{n+1} + \frac{a_{n-1}}{n}x^n + \ldots + a_0x) - \int \frac{1}{x}(\frac{a_n}{n+1}x^{n+1} + \frac{a_{n-1}}{n}x^n + \ldots + a_0x)\,dx$$

Taking x as a common factor,

$$\ln x\,(\frac{a_n}{n+1}x^{n+1} + \frac{a_{n-1}}{n}x^n + \ldots + a_0x) - \int (\frac{a_n}{n+1}x^n + \frac{a_{n-1}}{n}x^{n-1} + \ldots + a_0)\,dx$$

$$\ln x\ (\frac{a_n}{n+1}x^{n+1} + \frac{a_{n-1}}{n}x^n + \ldots + a_0x) - (\frac{a_n}{(n+1)^2}x^{n+1} + \frac{a_{n-1}}{n^2}x^n + \ldots + a_0x) + C$$

Example :Evaluate the following integral

$$\int (x^2 + 2x - 5)\ln x\,dx$$

signs	differentiation	integration
+	$\ln x$	$(x^2 + 2x - 5)$
-	$\frac{1}{x}$ $+\int$	$\frac{x^3}{3} + x^2 - 5x$

$$\int x^2 + 2x - 5)\ln x dx = \left(\frac{x^3}{3} + x - 5x\right)\ln x - \left(\frac{x^3}{9} + \frac{x^2}{2} - 5x\right) + C$$

Case 4 The product of two trigonometric functions such as

$$\int \sin(kx)\cos(mx)dx$$

signs	differentiation	integration
+	$\sin kx$	$\cos mx$
-	$k\cos kx$	$\frac{\sin mx}{m}$
+	$k^2 \sin kx$ $+\int$	$-\frac{\cos mx}{m^2}$

$$\int \sin(kx)\cos(mx)dx = \sin kx\,\frac{\sin mx}{m} + k\cos kx\,\frac{\cos mx}{m^2} - \int k^2 \sin kx\,\frac{\cos mx}{m^2}\,dx$$

$$\int \sin(kx)\cos(mx)dx + \int k^2 \sin kx\,\frac{\cos mx}{m^2} = \sin kx\,\frac{\sin mx}{m} + k\cos kx\,\frac{\cos mx}{m^2}$$

$$\int \sin(kx)\cos(mx)dx = \frac{1}{\left(1 + \frac{k^2}{m^2}\right)}\left(\sin kx\,\frac{\sin mx}{m} + k\cos kx\,\frac{\cos mx}{m^2}\right) + C$$

$$\left(\frac{m^2 + k^2}{k^2}\right)\sin kx\,\frac{\sin mx}{m} + k\cos kx\,\frac{\cos mx}{m^2} + C$$

Case 5 The product of inverse Trigonometric functions and polynomials

$$\int p_n(x)(\tan^{-1} x)dx$$

Consider the example $\int x^2 \tan^{-1} x\, dx$

Sings	differentiation	integration
+	$\tan^{-1}(x)$	x^2
-	$\frac{1}{x^2+1}$	$\frac{x^3}{3}$

The final set up for the problems

$$\int x^2 \tan^{-1} x dx = \frac{x^3}{3}\tan^{-1} x - \int \frac{x3}{3\left(x^2+1\right)} = \frac{x^3}{3}\tan^{-1} x - \frac{\ln\left(x^2+1\right)}{6} + \frac{x^2}{6} + C$$

Note that $\dfrac{x^3}{x^2+1} = x - \dfrac{x}{x^2+1}$

In general we can generalize the forms of any product of functions as

Signs	Derivative	antiderivative
+	F	G
-	$F^{(1)}$	$G^{(-1)}$
+	$F^{(2)}$	$G^{(-2)}$
-	$F^{(3)}$	$G^{(-3)}$
	$\vdots$	$\vdots$
$(\ 1)^{n}$	$F^{\ n}$	$G^{(-n)}$
$(\ 1)^{n+1}$	F^{n+1}	$G^{(-n-1)}$

Where (n) in column #1 stands for successive derivative, and (–n) in column #2 stands for successive anti-derivative So the final answer of such problem is

$$\int F(t)G(t)dt = FG^{-1} - F^{(1)}G^{(-2)} + F^{(2)}G^{(-3)} - \cdots + (-1)^n F^{(n)}G^{(-n-1)} + (-1)^{n+1}\int F^{(n+1)}(t)G^{-n-1}(t) =$$

$$\sum_{n=0}^{n}(-1)^k F^{(k)}G^{(-k-1)} + (-1)^{n+1}\int F^{(n+1)}(t)G^{(-n-1)}(t)dt$$

Tabular integration by parts and proving theorms

1) Laplace Transformation.(computation involving $L\{f(t)\} = -\int_0^\infty e^{-st}f(t)dt$ often requires several integration by parts.

Tabular integration by parts can be used to establish the fundamental formula for Laplace Transform for the n[th] derivative of a function.

Theorem let n be a positive integer and suppose that f(t) is a function such that $f^{(n)}(t)$ is piece wise continuous on the interval [0, ∞)] and suppose that there exist constants A ,b and M such that $\left|f^{(k)}(t)\right| \le Ae^{bt}$ if t≥ M for all k = 0,1,2,…., n-1 the

$$Ll\{f^{(n)}(t)\} = -f^{(n-1)}(0) - sf^{(n-2)}(0) - \cdots - s^{n-1}f(0) + s^n L\{f(t)\}$$

Proof

signs	Differentiation	integration
+	e^{-st}	$f^{(n)}(t)$
-	$-se^{-st}$	$f^{(n-1)}(t)$
+	s^2e^{-st}	$f^{(n-2)}(t)$
$\vdots$		$\vdots$
$(-)^{n-1}$	$(-1)^{n-1}s^{n-1}e^{-st}$ $f^{n}(t)$	$f^{(1)}(t)$
$(-1)^{n}$	$(-1)^{n}s^{n}e^{-st}$	f(t)

$$L(f^{(n)}(t)) = [e^{-st} f^{(n-1)}(t) + se^{-st} f^{(n-2)}(t) + \cdots + s^{n-1} e^{-st} f(t)]_{t=0}^{t=\infty} + \int_0^\infty s^n e^{-st} f(t) dt\,]$$

$$= -f^{(n-1)}(0) - sf^{(n-2)}(0) - \cdots - s^{n-1} f(0) + s^n L\{f(t)\}.$$

1)Taylor Formula

Tabular integration by parts provides a straight forward proof of Taylor 's formula with integral remainder term .

Theorem: Suppose f(t) has n+1 continuous derivatives throughout an interval containing a . If x is any number in the interval then

$$f(x) = f(a) + f^{(1)}(a)(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \int_a^x \frac{f^{(n+1)}(t)}{n!}(x-t)^n dt \cdots (1)$$

signs	Derivative	Antiderivative
+	$-f^{(1)}(t)$	-1
−	$-f^{(2)}(t)$	$(x-t)$
+	$-f^{(3)}(t)$	$-\frac{(x-t)^2}{2!}$
	⋮	⋮
$(-1)^{n+1}$	$-f^{(n)}(t)$	$(-1)^n \frac{(x-t)^{n-1}}{(n-1)!}$
$(-1)^{n+2}$	$(-f^{(n+1)}(t)$	$(-1)^{n+1} \frac{(x-t)^n}{n!}$

$$\int_a^x \left[-f^{(1)}(t)\right]\left[-1\right] dt = \left[-f^{(1)}(t)(x-t) - \frac{f^{(2)}(t)}{2!}(x-t)^2 - \cdots - \frac{f^{(n)}(t)^n}{n!}(x-t)^n\right]_{t=a}^{t=x} + \int_a^x \frac{f^n(t)}{n!}(x-t)^n dt$$

Equation (1) follows immediately

3. Residue Theorem for Meromorphic Functions

Tabular integration by parts also can be applied to a complex line integral and can be used to prove some theorems in complex analysis. One of these theorems is the Residue Theorem for Meromorphic function

Theorem Suppose f(z) is analytic in $D=\{z:\mathbf{0} < \left|z \quad z_{\mathbf{0}}\right| < R\}$ and has a pole of order m at z_0 .Then if $\mathbf{0} < r < R$

$$\oint_{|z-z_0|} f(z)dz = \frac{2\pi i}{(m-1)!}\lim_{z\to z_0}\left[\frac{d^{m-1}}{dz^{m-1}}(z-z_0)^m f(z)\right] \qquad (1)$$

Proof

signs	Derivatives	Antiderivatives
+	$(z-z_0)^m f(z)$	$(z-z_0)^{-m}$
-	$\frac{d}{dz}(z-z_0)^m f(z)$	$\frac{(z-z_0)^{-m+1}}{m-1}$
+	$\frac{d^2}{dz^2}(z-z_0)^m f(z)$	$\frac{(z-z_0)^{-m+2}}{(m-1)(m-2)}$
	$\vdots$	$\vdots$
$(-1)^{m-1}$	$\frac{d^{m-1}}{dz^{m-1}}(z-z_0)^m f(z)$	$(-1)^{m-1}\frac{(z-z_0)^{-1}}{(m-1)!}$

$$SO \oint_{|z-z_0|=r} (z-z_0)^m f(z)(z-z_o)^{-m}dz =$$

$$\left[\sum_{k=0}^{m-2}\frac{(m-k-2)!(\frac{d^k}{dz^k}(z-z_0)^m f(z)}{(m-1)!(z-z_0)^{m-k-1}}\right]\downarrow_{|z-z_0|=r} + \frac{1}{(m-1)!}\oint_{|z-z_0|=r}\frac{\frac{d^{m-1}}{dz^{m-1}}(z-z_0)^m f(z)}{z-z_0}dz$$

……………..(2)

Where $\left[\sum_{k=0}^{m-2}\frac{(m-k-2)!(\frac{d^k}{dz^k}(z-z_0)^m f(z)}{(m-1)!(z-z_0)^{m-k-1}}\right]$ must be evaluated for a closed circle $\left|z-z_0\right|=r$

And each function of column# 2 has a removable singularity at z_0 which means its single valued in D And each function in column #3 is a single valued function in D too Thus the summation term in equation (2) must vanish .The integral term on the right hand side of equation (2) is evaluated by Cauchy integral formula to get equation (1)

Tabulated integral and product of three more functions

$$\int xe^x \sin x dx$$

Stage 1

sings	Derivative	Antiderivative
+	xe^{x}	$\sin x$
-	$xe^{x}+e^{x}$	$-\cos x$
+	$xe^{x}+2e^{x}$	$-\sin x$
Stag2	-	
+	e^{x}	$\sin x$
-	e^{x}	$-\cos x$
+	e^{x}	$-\sin x$

$$\int xe^{x}\sin xdx = -xe^{x}\cos x + (xe^{x}+e^{x})\sin x - \int (xe^{x}+2e^{x})\sin xdx$$

$$\int xe^{x}\sin xdx = -xe^{x}\cos x + (xe^{x}+e^{x})\sin x - \int (xe^{x}\sin xdx) + \int 2e^{x}\sin xdx$$

$$\int xe^{x}\sin xdx + \int xe^{x}\sin xdx = -xe^{x}\cos x + (xe^{x}+e^{x})\sin x - \int 2e^{x}\sin dx$$

$$2\int xe^{x}\sin xdx = -xe^{x}\cos x + (xe^{x}+e^{x})\sin x + (-2\boxed{\int e^{x}\sin xdx})$$

$$\boxed{-2\int e^{x}\sin xdx =} - 2(-e^{x}\cos x + e^{x}\sin - \int e^{x}\sin xdx)$$

$$= -4\int e^{x}\sin dx = -2(-e^{x}\cos x + e^{x}\sin x)$$

$$\therefore \int e^{x}\sin xdx = \frac{1}{2}\left(e^{x}\sin x - e^{x}\cos x\right)$$ **Then,**

$$2\int xe^{x}\sin xdx = -xe^{x}\cos x + (xe^{x}+e^{x})\sin x + (e^{x}\sin x - e^{x}\cos x) + c$$

$$\int xe^{x}\sin xdx = \frac{1}{2}(-xe^{x}\cos x + (xe^{x}+e^{x})\sin x + (e^{x}\sin x - e^{x}\cos x)) + C$$

$$\int xe^{x}\sin xdx = \frac{1}{2}(xe^{x}(\sin x - \cos x) + 2e^{x}\sin x + -e^{x}\cos x) + C$$

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