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RESEARCH ARTICLE

THE EFFECT OF GRAVITY INCLINATION ON THE ESTIMATION OF CRITICAL PERMEABILITY IN A 2D – BIO-POROUS CONVECTION (BPC)

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ABSTRACT

The present investigation was carried out with the purpose of analyzing the 2D-stability analysis of bio-porous convection (BPC) in a suspension of motile microorganisms in a gravity inclined environment. Due to the extreme complexity of the problem the computational tools like Maple and Mathematica were used to get the analytic expressions. It is a well known fact that permeability of the porous medium is an important factor in the study of bioconvection. In the absence of gravity inclination, the results obtained were quite simple when compared to those of the inclined environment. It was found that the criterion for the existence of critical permeability was dominated by five parameters viz, cell eccentricity, gravity inclination, average swimming velocity, vertical disturbance and fluid velocity. The profiles of critical permeability vs cell eccentricity exhibited amazingly interesting features. The results were found to be in excellent agreement with the available results for the limiting cases.

INTRODUCTION

The phenomenon of bioconvection patterns in suspensions of swimming cells have been observed several decades ago. Ever since common algae, such as *Chlamydomonas nivalis*, *Euglena viridis*, *Crypthecodinium cohnii* and the ciliated protozoan *Tetrahymena pyriformis* were isolated, plumes of aggregating cells have been noticed in the culturing flasks. Platt (1961) coined the term ''bioconvection'' to describe the phenomenon of pattern formation in shallow suspensions of motile micro-organisms at constant temperature, on par with those found in convection experiments. However, this is by no means the first documented observation, which goes back to at least 1848 e.g., Wager (1911). Other experimental investigators are, Loeffer and Mefferd (1952) Nultsch and Hoff (1972) Plesset and Winet (1974) and, more recently, Kessler, (1984), (1985b), Bees (1996) and Bees and Hill, (1998), Levandowsky *et al.* (1975), Childress *et al.* (1975) etc. Hydrodynamic flows orient and convect. Bioconvection is therefore an exciting, complex, yet experimentally tractable, approach for studying the mutual interdependence of physics and biology, where the overall phenomenology greatly exceeds its primitive components (Kessler, 1989; Kessler and Hill, 1997).

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In most natural aquatic ecosystems, microorganisms are advected more or less passively in the large-scale flows generated by various imbalances of the environment. In a quiescent fluid, however, even their slow motion (typically a couple of meters per day) and physical interactions can result in considerable spatial rearrangements, and influence the dynamics of the system (Mendelson, 1999). Bioconvection is one of the oldest documented collective behaviours of independent microorganisms (Wager, 1911; Loeffer and Mefferd, 1952; Platt, 1961; Plesset and Winet, 1974), arising spontaneously in suspensions of diverse swimming microorganisms such as bacteria, algae or ciliated protozoa (Kessler, 1985; Pedley and Kessler, 1992; Kessler and Wojciechowski, 1997). Typically, bioconvective pattern formation occurs in shallow suspensions at high cell concentration if the density of cells is 5–15% larger than that of water and the average direction of the microorganisms' swimming is upward in response to some external stimulus, e.g. gravity, light or oxygen-concentration gradient. For instance, in non-aerated suspensions of *B. subtilis* (aerob, soil-living bacteria) upward swimming is a chemotactic behaviour directed by the oxygen gradient (Taylor *et al.,* 1999). Bioconvection is the name given to pattern formation in suspensions of microorganisms, such as bacteria and algae, due to up-swimming of the micro-organisms (Pedley and

Kessler 1992). Bioconvection has been observed in several bacterial species, including aerobic, anaerobic, and magnetotactic organisms, as well as in algal and protozoan cultures (Kessler and Hill (1997)). All have in common the sudden appearance of a pattern when viewed from above. In all cases the microorganisms are denser than water and on average they swim upwards (although the reasons for up-swimming may be different for different species). The algae (e.g. Chlamydomonas) are approximately 5% denser than water, whereas the bacteria are nearly 10% denser than water. Microorganisms respond to certain stimuli by tending to swim in particular directions. These responses are called taxes, examples being gravitaxis, phototaxis, chemotaxis and gyrotaxis. Gravitaxis indicates swimming in the opposite sense to gravity, phototaxis denotes swimming towards or away from light, and chemotaxis corresponds to swimming up chemical gradients. Gyrotaxis is swimming directed by the balance between the torque due to gravity acting on a bottom-heavy cell and the torque due to viscous forces arising from local shear flows. This paper was concerned with the 2D stability analysis of bioconvection in a suspension of motile gyrotactic microoganisms in a fluid saturated porous medium subject to gravity inclination. The method employed was perturbation technique and results were obtained by using the computational tools viz.com *Maple and Mathematica.* The effect of gravity inclination on critical permeability was studied and the results were studied through graphs.

MATERIALS AND METHODS

In this section the mathematical formulation of the problem together with the stability analysis were discussed .

Mathematical fomulation and analysis

The major assumptions utilized in this paper were; (i) the porous matrix does not absorb microorganisms. (ii) the suspension was dilute.(iii) the medium was an isotropic fluid saturated porous medium of uniform porosity (iv) no macroscopic motion of the fluid occurs (v) all the microoganisms were swimming vertically upwards. The governing equations for a two dimensional unsteady flow in a porous medium were obtained by volume averaging the equations of Pedley et al., [1988] model, utilizing the volume averaging procedure described in Whitaker [1999]. This procedure resulted in the replacement of the Laplacian viscous terms with the Darcian terms that described viscous resistance in a porous medium (Nield and Bejan [1999]).The whole system was inclined at an angle ∂ to the vertical. The governing equations were :

the momentum equation in the component form;

$$C_a \rho_o \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} - \frac{\mu v}{K} - n\theta\Delta\rho g \cos\partial \quad ,.(1)$$

$$\frac{\partial p}{\partial y} \; by \; \frac{\partial p}{\partial x}, \quad .(2)$$

the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad ..(3)$$

the conservation of cells

$$\frac{\partial n}{\partial t} = -div(nV + nW_c\hat{p} - D\Delta_n) \quad .(4)$$

where C_a was the acceleration coefficient ; ∂ was the angle of inclination to the vertical; D is the diffusivity of the microorganisms (this assumes that all random motion of the microorganisms can be approximated by a diffusive process); g was the gravitational acceleration; K was the permeability of the porous medium; n_o was the number density of microorganisms in the basic state; p was the excess pressure(above hydrostatic); $\hat{p}$ was the unit vector indicating the direction of swimming velocity of microorganisms ; t was the time; u, and v were the x-, and y-velocity components respectively;V was the velocity vector,(u,v); $W_c\hat{p}$ was the vector of average swimming velocity of microorganisms relative to the fluid(w_c is assumed to be constant); x and y were the Cartesian coordinates(y was the vertical coordinate); $\Delta\rho$ was the density difference $\rho_{cell} - \rho_0$; θ was the average volume of microorganisms ; μ was the dynamic viscosity, assumed to be approximately the same as that of water; ρ_o was the density of water.

Stability analysis

In order to obtain the stability criterion the perturbations of cell concentration, fluid velocity components and the unit vector $\hat{p}$ that indicates the direction of bacterial swimming were introduced as follows;

$$[n,u,v,\hat{p}](t,x,y)=[n_0+\varepsilon n',\varepsilon u',\varepsilon v',n'+\hat{p}'](t,x,y)\} \quad ...(5)$$

Where $\hat{k}$ was the vector in the vertically upwards y-direction, a prime denoted a perturbation quantity and ε was the small perturbation amplitude After eliminating the pressure , the perturbations (5) were substiiuted in to the governing equations which resulted in the following set of linearized equations:

$$c_a\rho_o \frac{\partial}{\partial t}\left(\frac{\partial u'}{\partial y} - \frac{\partial v'}{\partial x}\right) = -\frac{\mu}{K}\left(\frac{\partial u'}{\partial y} - \frac{\partial v'}{\partial x}\right) + \theta\Delta\rho g\left(\cos\partial \frac{\partial n'}{\partial x} - \sin\partial \frac{\partial n'}{\partial y}\right) \quad ..(6)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0 \quad ..(7)$$

$$\frac{\partial n'}{\partial t} = -div[n_o(V^1 + W_c\hat{p}') + n'w_c\hat{k} - D\nabla n') \quad ..(8)$$

where V^1 was the vector composed of perturbations of the corresponding velocity components,. (u',v') Pedley *et al.* [1988] analysed the impact of the gyrotaxes on the direction of swimming of microorganisms. Then obtained an equation that relates the perturbation of the swimming direction. $\hat{p}^1$ with perturbation of velocity components. For a two dimensional problem, the results of Pedley *et al*. [1988] can be presented as

$$\hat{p}^1 = (-B\xi, 0) \quad ...(9)$$

Where B was the time scale for the reorientation of microorganisms by the gravitational torque against viscous resistance. In Pedley and Kessler [1987] this parameter is called the "gyrotactic orientation parameter", It can be expressed as

$$B = \frac{\alpha_{\perp}\mu}{2h\rho_o g} \qquad ..(10)$$

where $\alpha_{\perp}$ was the dimensionless constant relating viscous torque to the relative angular velocity of the cell and h was the displacement of centre of mass of the cell from the centre of buoyancy. The components ξ of vector $\hat{p}^1$ in Eq.(9) was connected to perturbations of velocity component by the following equation:

$$\xi = (1+\alpha_o)\frac{\partial u^1}{\partial y} - (1-\alpha_o)\frac{\partial v^1}{\partial x} \qquad ..(11)$$

α_o is the cell eccentricity which is given by the following equation.

$$\alpha_o = \frac{a^2 - b^2}{a^2 + b^2} \qquad .(12)$$

Where a and b are the semi-major and semi-minor axes of the spheroidal cell respectively. By substituting Eq,(9) into Eq.(8) and accounting (11), the following equation for n' was obtained.

$$\frac{\partial n^1}{\partial t} + W_c\frac{\partial n^1}{\partial y} + W_c B n_o\left(\frac{\partial \xi}{\partial x}\right) = D\nabla^2 n^1 \qquad ...(13)$$

In order to study the stability of the system, the perturbation quantities were introduced in terms of individual fourier by Fourier modes:

$$[n', v'](t,x,y)=[N,U]\exp[\sigma t + i(lx+my)] \qquad ..(14)$$

Again by substituting Eq.(14) into continuity equation for perturbation quantities (7) the following equation for u' was obtained.

$$u'(t,x,y) = -\frac{Um}{l}\exp[\sigma t + i(lx+my)] \qquad ..(15)$$

In order to determine the amplitude equations for U and N Eqs .(14) and (15) were substituted into (6) and (13) which resulted in the following equations;.
which

$$U = \frac{-gKlN\Delta\rho\theta(l\cos\partial - m\sin\partial)}{k^2(\mu + C_a\rho_o K\sigma)} \qquad ..(16)$$

In a similar way, we get after simplification,

$$N\sigma + w_c Nim + w_c UBn_o\left[(1+\alpha_o)m^2 + (1-\alpha_o)l^2\right] = -DNk^2 \qquad ..(17)$$

where $k^2 = l^2 + m^2$. Eliminating the amplitudes from Eqs.(16) and (17) resulted in the following dispersion equation for the growth rate parameter σ :

$$k^2 C_a\rho_o K\sigma^2 + (Dk^4 C_a\rho_o K + k^2 C_a\rho_o Kimw_c + k^2\mu)\sigma + imw_c k^2\mu + k^4\mu D - Bn_o gKlw_c\Delta\rho\theta l [m^2(1+\alpha_o) + l^2(1-\alpha_o)](l\cos\partial - m\sin\partial) = 0 \qquad ..(18)$$

The two roots for the growth rate parameter σ computed from Eq.(18)were σ

$$\frac{1}{2}(-DC_a\rho_o Kk^3 - kimW_c C_a\rho_o K - k\mu + \text{sqrt}(D^2 C_a^2\rho_o^2 K^2 k^6$$
$$+2DC_a^2\rho_o^2 K^2 k^4 imW_c - 2DC_a\rho_o Kk^4\mu + k^2 i^2 m^2 W_c^2 C_a^2\rho_o^2 K^2$$
$$-2k^2 imW_c C_a\rho_o K\mu + k^2\mu^2 - 4C_a\rho_o K^2 Bn_o gl\Delta\rho\theta W_c m^3\alpha_o\sin(\delta)$$
$$-4C_a\rho_o K^2 Bn_o gl\Delta\rho\theta W_c m^3\sin(\delta)$$
$$+4C_a\rho_o K^2 Bn_o gl^2\Delta\rho\theta W_c m^2\alpha_o\cos(\delta)$$
$$+4C_a\rho_o K^2 Bn_o gl^3\Delta\rho\theta W_c\alpha_o m\sin(\delta)$$
$$+4C_a\rho_o K^2 Bn_o gl^2\Delta\rho\theta W_c m^2\cos(\delta) - 4C_a\rho_o K^2 Bn_o gl^3\Delta\rho\theta W_c m\sin(\delta)$$
$$-4C_a\rho_o K^2 Bn_o gl^4\Delta\rho\theta W_c\alpha_o\cos(\delta) + 4C_a\rho_o K^2 Bn_o gl^4\Delta\rho\theta W_c\cos(\delta)))$$
$$/(C_a\rho_o Kk)$$

and

$$\frac{1}{2}(-DC_a\rho_o Kk^3 - kimW_c C_a\rho_o K - k\mu - \text{sqrt}(D^2 C_a^2\rho_o^2 K^2 k^6$$
$$+2DC_a^2\rho_o^2 K^2 k^4 imW_c - 2DC_a\rho_o Kk^4\mu + k^2 i^2 m^2 W_c^2 C_a^2\rho_o^2 K^2$$
$$-2k^2 imW_c C_a\rho_o K\mu + k^2\mu^2 - 4C_a\rho_o K^2 Bn_o gl\Delta\rho\theta W_c m^3\alpha_o\sin(\delta)$$
$$-4C_a\rho_o K^2 Bn_o gl\Delta\rho\theta W_c m^3\sin(\delta)$$
$$+4C_a\rho_o K^2 Bn_o gl^2\Delta\rho\theta W_c m^2\alpha_o\cos(\delta)$$
$$+4C_a\rho_o K^2 Bn_o gl^3\Delta\rho\theta W_c\alpha_o m\sin(\delta)$$
$$+4C_a\rho_o K^2 Bn_o gl^2\Delta\rho\theta W_c m^2\cos(\delta) - 4C_a\rho_o K^2 Bn_o gl^3\Delta\rho\theta W_c m\sin(\delta)$$
$$-4C_a\rho_o K^2 Bn_o gl^4\Delta\rho\theta W_c\alpha_o\cos(\delta) + 4C_a\rho_o K^2 Bn_o gl^4\Delta\rho\theta W_c\cos(\delta)))$$
$$/(C_a\rho_o Kk) \qquad ....(19)$$

In order to prove that critical permeability exists it was absolutely necessary to prove that (i) the system was stable, when the permeability of the porous medium is close to zero and (ii) the system becomes unstable when the permeability was sufficiently large. Accordingly instability appeared only when the real part of σ was positive. It was observed that since the root with the positive sign in front of the second term in Eq.(19) of first root had a greater real part, analysis was concentrated on this root it self. The Taylor series expansion of this root about the point K=0 was found . By neglecting the quadratic and higher order terms in this expansion, the following solution (valid only for the small values of permeability) was obtained;

$\sigma =$

$$\frac{1}{2}\,\frac{-k\mu + \sqrt{k^2\mu^2}}{C_a\rho_o k}K^{-1} +$$

$$\frac{1}{2}\,\frac{-DC_a\rho_o k^3 - kimW_c C_a\rho_o + \dfrac{\frac{1}{2}\sqrt{k^2\mu^2}\,(-2k^2 imW_c C_a\rho_o\mu - 2DC_a\rho_o k^4\mu)}{k^2\mu^2}}{C_a\rho_o k}$$

$$+\frac{1}{2}\sqrt{k^2\mu^2}\Bigg(\frac{1}{2}(k^2 i^2 m^2 W_c^2 C_a^2\rho_o^2 + D^2 C_a^2\rho_o^2 k^6 + 2DC_a^2\rho_o^2 k^4 imW_c$$
$$-4C_a\rho_o Bn_o gl\Delta\rho\theta W_c m^3\alpha_o\sin(\delta) - 4C_a\rho_o Bn_o gl\Delta\rho\theta W_c m^3\sin(\delta)$$

$$+4C_a\rho_o Bn_o g l^2\Delta\rho\theta W_c m^2\alpha_o\cos(\delta)+4C_a\rho_o Bn_o g l^3\Delta\rho\theta W_c\alpha_o m\sin(\delta)$$
$$+4C_a\rho_o Bn_o g l^2\Delta\rho\theta W_c m^2\cos(\delta)-4C_a\rho_o Bn_o g l^3\Delta\rho\theta W_c m\sin(\delta)$$
$$-4C_a\rho_o Bn_o g l^4\Delta\rho\theta W_c\alpha_o\cos(\delta)+4C_a\rho_o Bn_o g l^4\Delta\rho\theta W_c\cos(\delta))/($$
$$k^2\mu^2)-\frac{1}{8}\frac{(-2k^2 i m W_c C_a\rho_o\mu-2DC_a\rho_o k^4\mu)^2}{k^4\mu^4}\Bigg)/(C_a\rho_o k)K+O(K^2)$$
..(20)

From the above Eq. it was observed that for K=0, σ had a negative real part. $-k^2D$ which was independent of ∂. This suggested that for sufficiently small values of permeability the system was stable. Therefore in order to prove that the system becomes unstable with the increase K, it was absolutely necessary to show that the real part of σ would be positive. Suppose m=o (which corresponded to the case of no vertical disturbances). In this case the root of the Eq. (20) with the greater real part was found to be

$\sigma =$

$$\frac{1}{2}\frac{-k\mu+\sqrt{k^2\mu^2}}{C_a\rho_o kK}+\frac{\frac{1}{2}\left(-DC_a\rho_o k^3-\frac{\sqrt{k^2\mu^2}\,DC_a\rho_o k^2}{\mu}\right)}{C_a\rho_o k}+\frac{1}{2}\sqrt{k^2\mu^2}\Bigg(\frac{1}{2}(D^2C_a^2\rho_o^2k^6$$
$$-4C_a\rho_o Bn_o g l\Delta\rho\theta W_c m^3\alpha_o\sin(\delta)-4C_a\rho_o Bn_o g l\Delta\rho\theta W_c m^3\sin(\delta)$$
$$+4C_a\rho_o Bn_o g l^2\Delta\rho\theta W_c m^2\alpha_o\cos(\delta)+4C_a\rho_o Bn_o g l^3\Delta\rho\theta W_c\alpha_o m\sin(\delta)$$
$$+4C_a\rho_o Bn_o g l^2\Delta\rho\theta W_c m^2\cos(\delta)-4C_a\rho_o Bn_o g l^3\Delta\rho\theta W_c m\sin(\delta)$$
$$-4C_a\rho_o Bn_o g l^4\Delta\rho\theta W_c\alpha_o\cos(\delta)+4C_a\rho_o Bn_o g l^4\Delta\rho\theta W_c\cos(\delta))/($$
$$k^2\mu^2)-\frac{1}{2}\frac{D^2C_a^2\rho_o^2k^4}{\mu^2}\Bigg)K/(C_a\rho_o k)$$
....(21)

The upper limit of the critical permeability is computed by solving the equation (21) for σ =0

$$K_{crit}^{upper}=-\frac{D\mu}{Bn_o g\Delta\rho\theta W_c\cos(\delta)(\alpha_o-1)}$$
…(22)

The above equation clearly proved and predicted that the existence of critical permeability and further it was observed that the upper limit of the observed that the above solution coincides with that of uninclined case (Kuznetsov (2002)) in the limit ∂ =0. The critical permeability was the minimum value of K for all allowable wavenumbers and .critical permeability was more when compared to the uninclined case (∂ =0).

Estimation of the value of critical permeability

For this purpose the linearized, Eq. (20) was considered and from the of Re(σ)=0, σ was computed for small values of criticalpermeability;

$$\frac{1}{2}\frac{-k\mu+\sqrt{k^2\mu^2}}{C_a\rho_o kK}+\frac{\frac{1}{2}\left(-DC_a\rho_o k^3-\frac{\sqrt{k^2\mu^2}\,DC_a\rho_o k^2}{\mu}\right)}{C_a\rho_o k}+\frac{1}{2}\sqrt{k^2\mu^2}\Bigg(\frac{1}{2}(D^2C_a^2\rho_o^2k^6$$
$$-4C_a\rho_o Bn_o g l\Delta\rho\theta W_c m^3\alpha_o\sin(\delta)-4C_a\rho_o Bn_o g l\Delta\rho\theta W_c m^3\sin(\delta)$$
$$+4C_a\rho_o Bn_o g l^2\Delta\rho\theta W_c m^2\alpha_o\cos(\delta)+4C_a\rho_o Bn_o g l^3\Delta\rho\theta W_c\alpha_o m\sin(\delta)$$
$$+4C_a\rho_o Bn_o g l^2\Delta\rho\theta W_c m^2\cos(\delta)-4C_a\rho_o Bn_o g l^3\Delta\rho\theta W_c m\sin(\delta)$$
$$-4C_a\rho_o Bn_o g l^4\Delta\rho\theta W_c\alpha_o\cos(\delta)+4C_a\rho_o Bn_o g l^4\Delta\rho\theta W_c\cos(\delta))/($$
$$k^2\mu^2)-\frac{1}{2}\frac{D^2C_a^2\rho_o^2k^4}{\mu^2}\Bigg)K/(C_a\rho_o k)=0$$
..(.23)

solving this equation for $\hat{K}$ results in

$$\hat{K}=$$
$$-D(m^2+l^2)^2\mu/(Bn_o g l\Delta\rho\theta W_c(m^3\alpha_o\sin(\delta)+m^3\sin(\delta)-lm^2\alpha_o\cos(\delta)$$
$$-l^2\alpha_o m\sin(\delta)-lm^2\cos(\delta)+l^2 m\sin(\delta)+l^3\alpha_o\cos(\delta)-l^3\cos(\delta)))$$
..(24)

Investigation of function $\hat{K}$ (l, m, ∂) for extremum gives two equations which are identical:

$$\frac{\partial\hat{K}}{\partial l}=0=\frac{\partial\hat{K}}{\partial m}=$$
$$-D(m^2+l^2)\mu m(2m^3 l\cos(\delta)-m^4\alpha_o\sin(\delta)+l^4\sin(\delta)+2l^3 m\cos(\delta)$$
$$+2m^3 l\alpha_o\cos(\delta)-l^4\alpha_o\sin(\delta)-6l^3 m\alpha_o\cos(\delta)-m^4\sin(\delta)+6l^2m^2\alpha_o\sin(\delta)$$
$$)/(Bn_o g\Delta\rho\theta W_c(-m^3\alpha_o\sin(\delta)-m^3\sin(\delta)+lm^2\alpha_o\cos(\delta)+l^2\alpha_o m\sin(\delta)$$
..(25)

one of the feasible solution of (25) was;

m=

$$\frac{\cos\partial\, l}{2\sin\partial}-\frac{1}{2}\sqrt{\left[\frac{\cos\partial^2 l^2}{\sin\partial^2}+\frac{6l^2\alpha_o}{1+\alpha_o}-\frac{2\;l^2\sin\partial\,\alpha_o}{\sin\partial+\sin\partial\,\alpha_o}\right.}$$
$$+(4\;2^{1/3}(-\cos\partial^2 l^4-l^4\sin\partial^2+2\cos\partial^2 l^4\alpha_o+3\cos\partial^2 l^4\alpha_o^2+4l^4\sin\partial^2\alpha_o^2))/(\sin\partial(1+\alpha_o)$$
.....(26)

The full expression for m was found to be extremely lengthy and hence a part of it was presented. Substituting the positive root of m in (24) resulted in the expression for $\hat{K}$ .The computation was done through *Mathematica.* Because of its extreme length the result was not presented here.

$K_{crit}=$

$$[illegible]\; l^2 + \frac{\text{cosd}\,l}{2\,\text{sind}} - \frac{1}{2}\;[illegible]\;\frac{\text{cosd}^2 l^2}{\text{sind}^2}+\frac{6\,l^2 a_o}{1+a_o}-\frac{2\,l^2\,\text{sind}\,a_o}{\text{sind}+\text{sind}\,a_o}$$
$$+\;4\;2^{1\;3}\;[illegible]\;\text{cosd}^2\,l^4 - l^4\,\text{sind}^2+2\,\text{cosd}^2\,l^4\,a_o+3\,\text{cosd}^2\,l^4\,a_o^2$$
$$+4\,l^4\,\text{sind}^2 a_o^2\;[illegible]\;\text{sind}\;[illegible]\;+a_o\;[illegible]\;2\,l^6\,\text{sind}^3\,a_o^3$$
…..(27)

This expression for critical permeability was valid for the restricted values of α_0 and ∂ .The following points were observed in the present investigation.:

(i) $\frac{\partial\hat{K}}{\partial l}=0=\frac{\partial\hat{K}}{\partial m}$ results in a biquadratic equation when $\partial\neq 0$,and in such case, the solutions for l and m found by using the computational *Mathematica.* When ∂ =0, the solutions were extremely simple and were in excellent agreement with those of Kuznetsov & Avramenko (2002).

(ii) The computation of the critical permeability was extremely complex in the sense that K_{crit} strongly depends on these parameters viz., m, α_0 and ∂. The results were presented through graphs (Figures 1 to 9)

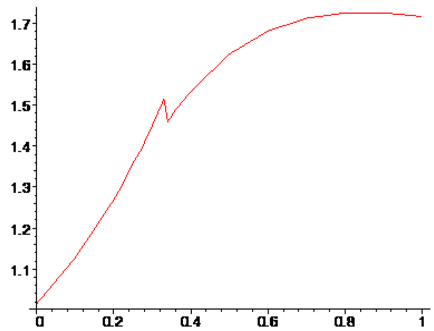

Fig1 dependence of K_{crit} on the α_o when $\delta = 10^o$

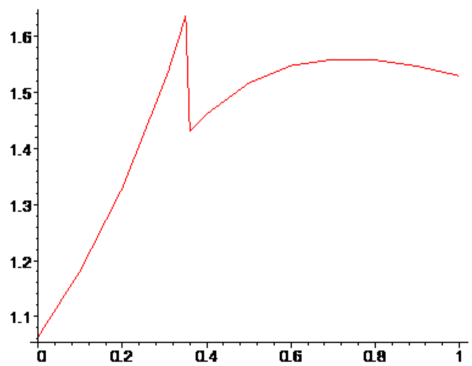

Fig2 dependence of K_{crit} on the α_o when $\delta = 20^o$

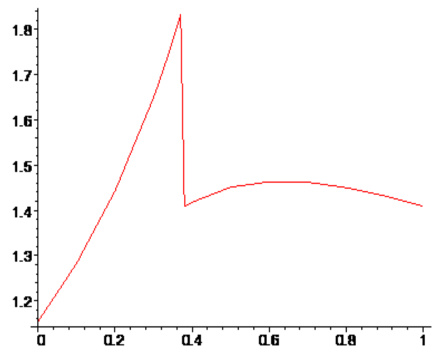

Fig3 dependence K_{crit} on the α_o when $\delta = 30^o$

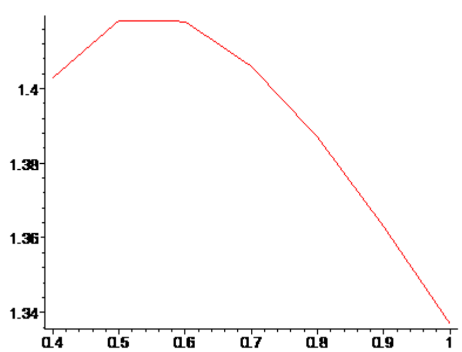

Fig4 dependence of K_{crit} on the α_o when $\delta = 40^o$

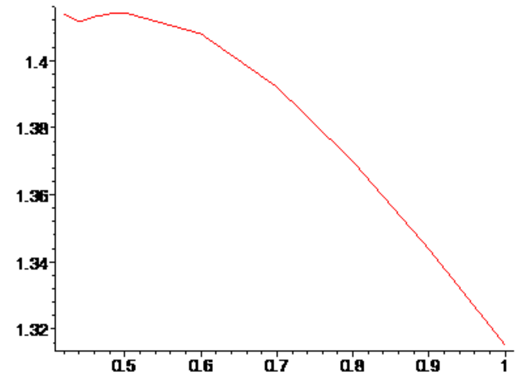

Fig5 dependence of K_{crit} on the α_o when $\delta = 45^o$

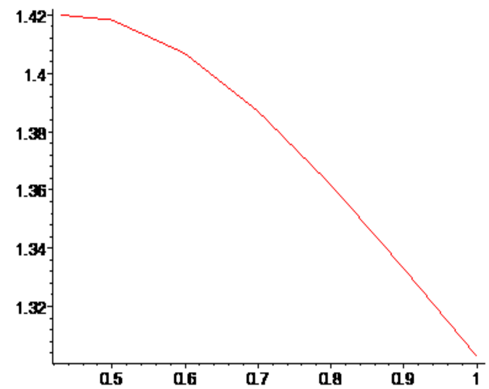

Fig6 dependence of K_{crit} on the α_o when $\delta = 50^o$

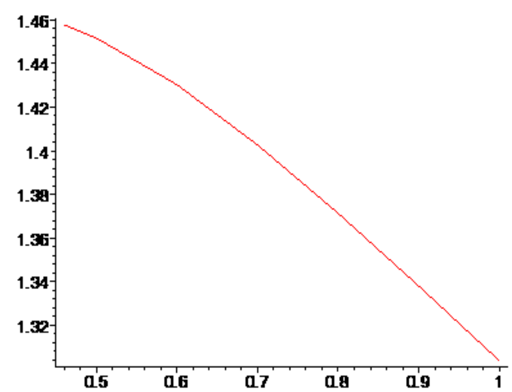

Fig7 dependence of K_{crit} on the α_o when $\delta = 60^o$

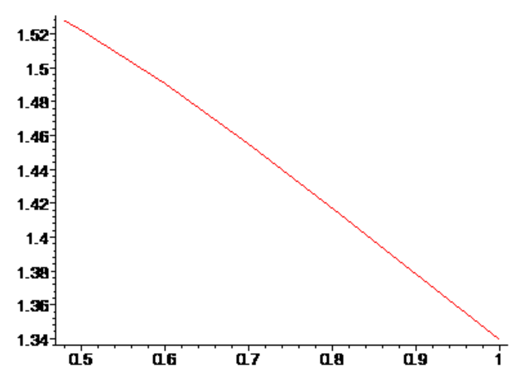

Fig8 dependence of K_{crit} on the α_o when $\delta = 70^o$

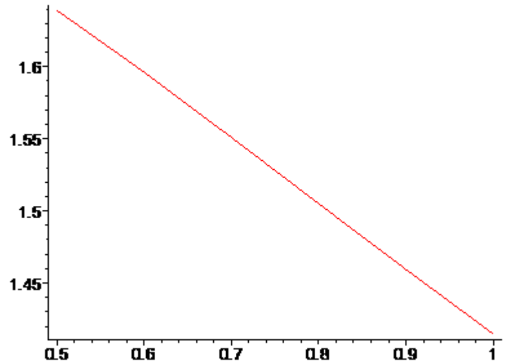

Fig9 dependence of K_{crit} *on the* α_o *when* $\delta = 80^o$

RESULTS AND DISCUSSION

A glance at Eq. (12) revealed that α_0 would change between zero and unity. When ∂=0, and $\alpha_0 \le 1/3$, the function $\hat{K}$ (l, m, ∂) defined by Eq. (24) did not possess an extremum. In that case the minimum value clearly occurred at the boundary of the domain, at m=0, which physically corresponded to the case of no vertical disturbances. In the absence of inclination(∂=0) the present investigation associated with bioconvection in a porous medium subject to gravity inclination yielded the expressions for the critical permeability as:

$$K_{crit} = \begin{vmatrix} \dfrac{1}{(1-\alpha_o)} \dfrac{D\mu}{Bn_o g W_c \Delta\rho\theta} \; for \; 0 \le \alpha_o \le 1/3 \\ \dfrac{8\alpha_o}{(1+\alpha_o)^2} \dfrac{D\mu}{Bn_o g W_c \Delta\rho\theta} \; for \; 1/3 \le \alpha_o \le 1 \end{vmatrix} \quad \ldots\ldots(26)$$

Which were exactly as those of Kuznetsov & Avramenko (2002). Thus the results of the present investigations were in excellent agreement with those of available theoretical results**.**

But when $\partial \ne 0$ the corresponding expression was found to be extremely lengthy and hence the predictions could be made only with the numerical computations. The one of the important observations was the extremely complex nature of the different resulting equations. In the absence of the inclination the demarcation value of α_0 was 1/3 and the K_{crit} was obtained under the assumption that the Taylor series expansion for σ contained only the linear terms.. This was clearly a good approximation when the diffusivity of cells D was small.The present investigation predicted the following important results:

(i) the critical permeability K_{crit} had a strong dependence on the cell eccentricity α_0 and gravity inclination ∂. Table1 clearly predicted the range of α_0 for m=o and $m \ne 0$ corresponding to each value of ∂, It was observed that for all values of ∂ the maximum values of K_{crit} were well with in the value 2.

(ii) the figures 1-9 predicted that the strong dependency of K_{crit} on α_0 and ∂.

(iii) in all cases (m=0) K_{crit} monotonically increases as α_0 increases from 0 to unity. However for values $m \ne 0$ only for very small values of ∂ $(0 \le \partial \le 10)$. However for large values ∂ the dependence of an α_0 was highly non-linear.

(iv) When α_0 increased to unity the microoganisms became more elongated.

(v) When the permeability was in the range of $0 \le K \le K_{crit}$, the system was stable. This suggested that the system with elongated microoganisms had a wider range of stability with respect K and ∂ than the system with spherical microoganisms .

(vi) another important finding was when the shape of the microoganisms is closed to spherical ($\alpha_0 \approx 0.33$) the most unstable disturbance were those with 0 vertical wave number and uninclined environment (m=0, ∂=0). This behaviour changes rapidly with ∂ (figures 1-9).

(vii) suppose the microoganisms were sufficiently elongated ($0.33 \le \alpha_o \le 1$) the most unstable disturbances were those with non-zero vertical wavenumber ($m \ne 0$; ∂=0 and $\partial \ne 0$).

(viii) in fact physically vertical disturbances were important for the suspension of elongated particles because the only vertical shear could orient the direction of swimming of elongated microoganisms away from vertical direction in the presence or absence of gravity inclination. This was the mechanism of gyrotaxis that would make the system unstable. It was observed that the gravity inclination played the dominant role on the mechanism of gyrotaxis associated with bioconvection in a porous medium in an inclined environment.

(ix) actually Pedley et al.,(1988) observed similar observations in the case of bioconvection in a fluid medium. But the present investigation very clearly predicted similar conclusions in the case bioconvection of in a porous medium in the presence and absence of gravity inclination. Therefore finally it was concluded that there existed (a) important similarities between bioconvection in porous and fluid media and (b) many dissimilarities between bioconvection in inclined and uninclined environments.

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