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RESEARCH ARTICLE

ASSESSING HIGH SCHOOL STUDENTS' MATHEMATICS COMPETENCY: CONSTRUCT SOME NEW RESULTS ON TRIANGLE

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ABSTRACT

In this paper, we help grade 10th Vietnamese students to construct some new results on triangle. Then we process to assess their mathematics competency.

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INTRODUCTION

How do you know if your students are achieving your specific learning goals for a course? Assessment is essential not only to guide the development of individual students but also to monitor and continuously improve the quality of programs. Competency evaluations of high school students provide excellent feedback about student satisfaction and teaching. In this paper, we use some examples in references (Nhi *et al*., 2013; 2015; 2012) to teaching the sudents.

Mathematics competency assessment of high school students in Vietnam

Level 1. The teachers assess students' competency of recalling knowledge

The students recall some formulas and apply them on the edges of a triangle.

In this paper, We denote: Given a triangles ABC with the edges $a = BC, b = CA, c = AB$. Denoted respectively O, R is the center and radius of circumcircle of triangle ABC, I, r are the centers and radius of incircle of circumcircle of triangle ABC, J_a, J_b, J_c are the centers of three escribed circles of triangle ABC have three center respectively being r_a, r_b, r_c. Denoted h_a, h_b, h_c are the lengths of altitudes respectively sides a, b, c of triangle ABC. Set $S = S_{ABC}$, $2p = a + b + c.$

Example 1 *With the above denoted, we have* a, b, c *are three solutions of the cubic polynomial* $x^3 - 2px^2 + (p^2 + r^2 + 4Rr)x - 4Rrp.$

Proof. The students given some formulas then changing them.

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From $tan\dfrac{A}{2}=\dfrac{r}{p-a}$ and $a=2R\sin A$, we obtain $a=2R\dfrac{2\tan(A/2)}{1+\tan^2(A/2)}$.

Thus, we obtain $a=4Rr\dfrac{p-a}{r^2+(p-a)^2}$ or $a^3-2pa^2+(p^2+r^2+4Rr)a-4Rrp=0.$

Similarly, we have $b^3-2pb^2+(p^2+r^2+4Rr)b-4Rrp=0$

and $c^3-2pc^2+(p^2+r^2+4Rr)c-4Rrp=0.$

Therefore, a,b and c are three solutions of the equation

$x^3-2px^2+(p^2+r^2+4Rr)x-4Rrp=0$

Level 2. The teachers assess students' competency at higher level as they give hypothesis and prove them.

From the example 1, the students construction new results.

Example 2 *With the above denoted.*

i) Construction the relation of h_a,h_b,h_c with the cubic polynomial

ii) Construction the relation of r_a,r_b,r_c with the cubic polynomial

Proof.

i) Using results example 1 we have a,b and c are the solutions of the equation $x^3-2px^2+(p^2+r^2+4Rr)x-4Rrp=0$ and $S=pr$, we deduce that $\dfrac{2S}{a},\dfrac{2S}{b}$ and $\dfrac{2S}{c}$ are the solutions of the equation

$$2S^2-\frac{2S^2}{r}y+\frac{\frac{S^2}{r^2}+4Rr+r^2}{2}y^2-Ry^3=0.$$

Hence, h_a,h_b,h_c are three solutions of the equation $x^3-\dfrac{S^2+4Rr^3+r^4}{2Rr^2}x^2+\dfrac{2S^2}{Rr}x-\dfrac{2S^2}{R}=0.$

ii) Because $\tan\dfrac{A}{2}=\dfrac{r_a}{p}$ and $a=2R\dfrac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$, we have $a=4Rr_a\dfrac{p}{r_a^2+p^2}$. Otherwise $S=r_a(p-a)=rp$, we deduce that $a=\dfrac{r_ap-S}{r_a}=\dfrac{(r_a-r)p}{r_a}$. Therefore,

$$4Rr_a\frac{p}{r_a^2+p^2}=\frac{(r_a-r)p}{r_a}\text{namely}(r_a-r)(r_a^2+p^2)=4Rr_a^2$$

that means r_a is a root of the equation $x^3-(4R+r)x^2+p^2x-p^2r=0$. Similarly, r_b and r_c are also solutions of the equation

$$x^3-(4R+r)x^2+p^2x-p^2r=0.$$

At the end of Level 2, The students understand knowledge and they give theorem 1.

Theorem 1 [4] *With the above denoted, the following holds*

i) a,b,c are three solutions of the cubic polynomial

$$x^3-2px^2+(p^2+r^2+4Rr)x-4Rrp. \qquad (1)$$

ii) h_a, h_b, h_c are three solutions of the cubic polynomial

$$x^3 - \frac{S^2 + 4Rr^3 + r^4}{2Rr^2}x^2 + \frac{2S^2}{Rr}x - \frac{2S^2}{R}. \qquad (2)$$

iii) r_a, r_b, r_c are three solutions of the cubic polynomial

$$x^3 - (4R+r)x^2 + p^2x - p^2r. \qquad (3)$$

Level 3. The teachers assess students' competency of mobilizing knowledge and creative thinking

The students use theorem 1 to prove some equalities and also present some new equalities that seem to be difficult if they are built from geometric properties.

Example 3 *From theorem 1, construction new results.*

i) From (3) of theorem 1, we have $x^3 - (4R+r)x^2 + p^2x - p^2r = (x-r_a)(x-r_b)(x-r_c)$.

Then, choosing $x = r$, we have $(r_a - r)(r_b - r)(r_c - r) = 4Rr^2$

Namely $(\frac{r_a}{r} - 1)(\frac{r_b}{r} - 1)(\frac{r_c}{r} - 1) = 4\frac{R}{r}$.

ii) Because r_a, r_b, r_c are three solutions of the equation (1), we have

$$\frac{1}{x-r_a} + \frac{1}{x-r_b} + \frac{1}{x-r_c} = \frac{3x^2 - 2(4R+r)x + p^2}{x^3 - (4R+r)x^2 + p^2x - p^2r}.$$

Choosing $x = r$, we obtain $\frac{1}{r_a - r} + \frac{1}{r_b - r} + \frac{1}{r_c - r} = \frac{r^2 - 8Rr + p^2}{4Rr^2}$. So

$$\frac{4R}{r_a - r} + \frac{4R}{r_b - r} + \frac{4R}{r_c - r} = 1 - 8\frac{R}{r} + \frac{r_a r_b r_c}{r^3}.$$

iii) Applying formulas 2, 3 of theorem 1 and Vieta's formula, we have

$$\frac{(r_a - r_b)^2}{r_a r_b} + \frac{(r_b - r_c)^2}{r_b r_c} + \frac{(r_c - r_a)^2}{r_c r_a} = \frac{(4R+r)p^2}{p^2 r} - 9 = \frac{4R}{r} - 8.$$

Thus, we get Proposition 1

Proposition 1. *With the above denoted, we have*

i) $(\frac{r_a}{r} - 1)(\frac{r_b}{r} - 1)(\frac{r_c}{r} - 1) = 4\frac{R}{r}$.

ii) $\frac{4R}{r_a - r} + \frac{4R}{r_b - r} + \frac{4R}{r_c - r} = 1 - 8\frac{R}{r} + \frac{r_a r_b r_c}{r^3}$.

iii) $\frac{(r_a - r_b)^2}{r_a r_b} + \frac{(r_b - r_c)^2}{r_b r_c} + \frac{(r_c - r_a)^2}{r_c r_a} = \frac{4R}{r} - 8$.

Example 4. *Given ABC with the above denoted. Construction the relation of h_a, h_b, h_c and R, r, r_a, r_b, r_c.*

i) From theorem 1 we have r_a, r_b, r_c are three solutions of the equation

$$x^3-(4R+r)x^2+p^2x-p^2r=0$$

we deduce that $\frac{1}{r_a},\frac{1}{r_b},\frac{1}{r_c}$ are three solutions of the equation

$$p^2rx^3-p^2x^2+(4R+r)x-1=0.$$

Using Vieta's formula, we have

$$\frac{1}{r_a^2}+\frac{1}{r_b^2}+\frac{1}{r_c^2}=(\frac{1}{r_a}+\frac{1}{r_b}+\frac{1}{r_c})^2-2(\frac{1}{r_a}\frac{1}{r_b}+\frac{1}{r_a}\frac{1}{r_c}+\frac{1}{r_b}\frac{1}{r_c})=\frac{1}{r^2}-2\frac{4R+r}{p^2r}.$$

Otherwise, applying formula (2) of theorem 1 and Vieta's formula, we have

$$4(\frac{1}{h_a^2}+\frac{1}{h_b^2}+\frac{1}{h_c^2})=\frac{2p^2-2r^2-8Rr}{p^2r^2}=\frac{2}{r^2}-2\frac{4R+r}{p^2r}.$$

Therefore, $4(\frac{1}{h_a^2}+\frac{1}{h_b^2}+\frac{1}{h_c^2})=\frac{1}{r_a^2}+\frac{1}{r_b^2}+\frac{1}{r_c^2}+\frac{1}{r^2}.$

ii) Applying theorem 1 and Vieta's formula, we obtain

$$h_ah_b+h_bh_c+h_ch_a=r\frac{2S^2}{Rr^2}=2r\frac{p^2}{R}=\frac{2r}{R}(r_ar_b+r_br_c+r_cr_a).$$

Thus, we get Proposition 2.

Proposition 2. *Given ABC with the above denoted. We have*

i) $4(\frac{1}{h_a^2}+\frac{1}{h_b^2}+\frac{1}{h_c^2})=\frac{1}{r_a^2}+\frac{1}{r_b^2}+\frac{1}{r_c^2}+\frac{1}{r^2}.$

ii) $h_ah_b+h_bh_c+h_ch_a=\frac{2r}{R}(r_ar_b+r_br_c+r_cr_a).$

At the end of Level 3, students' mathematics competence is good.

Level 4. The teachers assess students' self-deducing skill based on involving knowledge

i) Using result (2) of the theorem 1, we have $h_a^2+h_b^2+h_c^2\ge h_ah_b+h_bh_c+h_ch_a=\frac{2S^2}{Rr}$

deduce that $h_a^2+h_b^2+h_c^2\ge\frac{2S^2}{Rr}$. Furthermore $R\ge 2r$. Thus $h_a^2+h_b^2+h_c^2\ge\frac{4S^2}{R^2}$.

ii) In following equation: $y^3-\frac{S^2+4Rr^3+r^4}{2Rr^2}y^2+\frac{2S^2}{Rr}y-\frac{2S^2}{R}=(y-h_a)(y-h_b)(y-h_c)$

Let $y = r$ we have $(h_a - r)(h_b - r)(h_c - r) = \dfrac{S^2 + 2Rr^3 + r^4}{2R}$

From $R \geq r$ deduce that $$\frac{S^2 + 5r^4}{2R} \leq (h_a - r)(h_b - r)(h_c - r) \leq \frac{S^2 + 5r^4}{4r}.$$

iii) We have $$h_a h_b + h_b h_c + h_c h_a = r\frac{2S^2}{Rr^2} = 2r\frac{p^2}{R} = \frac{2r}{R}(r_a r_b + r_b r_c + r_c r_a).$$

Thus $h_a^2 + h_b^2 + h_c^2 \geq \dfrac{2r}{R}(r_a r_b + r_b r_c + r_c r_a).$

Thus, we get Corollary 1 .

Corollary 1. *Given triangle* ABC*, we have inequalities:*

i) $h_a^2 + h_b^2 + h_c^2 \geq \dfrac{4S^2}{R^2}.$

ii) $\dfrac{S^2 + 5r^4}{2R} \leq (h_a - r)(h_b - r)(h_c - r) \leq \dfrac{S^2 + 5r^4}{4r}.$

iii) $h_a^2 + h_b^2 + h_c^2 \geq \dfrac{2r}{R}(r_a r_b + r_b r_c + r_c r_a).$

At the end of Level 4, the students are at high level of self-learning.

Conclusion

In vietnam, students' competency assessment is limited. this paper present a method to assess students' mathematical competence in a specific mathematics knowledge. based on the results, teachers and students could improve their teaching and learning.

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