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RESEARCH ARTICLE

USING EXPERIMENTAL DESIGN PROCEDURE IN SOLVING LINEAR PROGRAMMING PROBLEM

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ABSTRACT

The study is about developing a class of line search exchange algorithm for solving linear programming problem using an experimental design procedure. The new algorithm, namely, minimum exchange algorithm has been developed and a first necessary condition for the existence of an optimizer of a linear programming problem has been obtained. This algorithm has been shown to converge. Numerical illustration and comparison show that the algorithm compares favourably with the simplex method for solving linear programming problems.

INTRODUCTION

The concept of an experiment is brought to focus in investigation to discover something about a particular process or to compare the effects of several conditions on some phenomenon. Experimental design refers to the process by which an experiment is planned so that the appropriate data are collected and analyzed using statistical methods to achieve valid and objective conclusion Montgomery (1976). The statistical approach to experimental design is necessary if we wish to draw meaningful conclusion from the data. Experimental design is concerned with detailed methods of carrying out an experiment in order to achieve maximum desired response objective. There are two aspects to any experimental problem, the design of the experiment and the statistical analysis of the data. These two subjects are closely related, since the method of the analysis depends directly on the design employed. In experimental design, we have three methods of design, namely, randomized design, factorial design and response surface design. In this study we are using the response surface design method in solving our problem

RESPONSE SURFACE METHODOLOGY

Response surface methodology [RSM] is a sequential procedure and it offers effective means for using experimental design principles to determine the optimizer of the real-valued function. Response surface methodology is a collection of mathematical and statistical techniques used in analyzing problems where several independent variables (or factors) influence the value of dependent response (F(x)) and the primary goal is to determine the value of the independent variable that maximize (or minimizes) the response. Response surface methodology is one of the important branches of experimental design and is a critical technology in developing new processes and optimizing their performance. Its objective is for quality improves including reduction of variability and improvment process and production performance. Onukogu (1997) states that response surface methodology can be seen as a bridge linking the subject of experimental design with the subject of our constrained optimization. Myers and Montgomery (1995). states that response surface is a mathematical and statistical techniques useful for optimizing the stochastic function. Response surface methodology is extremely useful as an automate tool for model calibration and validation especially for modern computational multi-agent large-scale social network system that are becoming heavily used in modeling and simulation. The most extensive application of the response surface methodology is in particular situation where several input variables potentially influence some performance measures or quality characteristics of the process. The performance of the quality characteristics is called the response. The input variables are sometimes called the independent variables and they are subjected to control by the experimenter. There is a problem faced by the experimenters in many fields where in general the response variables of interest is Y, and there is a set of predictor variables $X_1, X_2, \ldots, X_n$, identifying and fitting from experimental data an appropriate response surface model require some use of statistical experimental design fundamentals regression modeling techniques and optimization method. These three topics are usually combined into response surface methodology. In some response surface experiments there can be one or more linear dependence among regressor variables in the model.

The use of experimental design methods to solve unconstrained optimization problems is well established that the topic which is usually called Response Surface Methodology (RSM) has become a permanent feature in every important text on design of experiments. The aim is to determine the value of the independent variables that maximizes (or minimizes) the response. We denote the independent variable by $x_1, x_2, \ldots x_n$, and assume that these variables can be controlled by the experimenter with negligible error. The response function, f $(\underline{x})$, which is usually unknown, is represented by a regression function, $y(\underline{x})$, with additive error, thus we write.

$$Y(\underline{x}) = f(x_1, x_2, \ldots, x_n) + e \ldots(1)$$

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where e is the random error , f(x_1 , x_2, . . .,x_n) is called the response surface. The problem is to find the local optimizer, $\underline{x}^*$; i.e. The levels of the factor

$X^* = (x_1^*, x_2^*, \ldots, x_n^*)'$ such that for every $\underline{x}$

$$F(\underline{x}^*) \begin{cases} \geq f(x) \text{ if } f(x) \text{ is to be maximized} \\ \leq f(x) \text{ if } f(x) \text{ is to be minimized.} \end{cases}$$

The prime ()′ indicates the transpose of a vector or matrix x is any combination of the level of the factors. Besides locating the optimizer, response surface methodology also deals with exploration of the surface around the optimum point. The aim is to gain a better understanding of the optimal region and, if possible, determine the alternative optima that could be technically or economically more feasible than the one first found. The problem is to seek the various ways of locating the optimizer of response surface. Once this aspect of response surface methodology is well grounded then the problem of exploring within the optimal sub- region will be made easier. There are dozens of exploration techniques. The development and the application of these techniques are influenced by three general conditions.

- The nature of the response; whether it is nominal or quantitative.
- The factor or variates; whether they are quantitative or qualitative or mixed.
- The nature and magnitude of errors; whether it is largely systematic i.e. if it is due to randomness or bias, or both.

One of the difficulties with nominal response is that the search cannot ordinarily be gradient-based since the gradient is not defined; rather, the search is often directed along the path of increasing (or decreasing) response value. In this study we are using a line search algorithm in solving linear programming problem.

LINE SEARCH TECHNIQUES

The aim of any search scheme is to maximize performance .i.e. to locate the optimum of the response surface with the fewest number of iterations and minimum amount of computation at each iteration. To get this, it requires the best decision on.

1.The starting point of the sequence.
2.The regression model
3.The design of experiments
4.The direction of the search.
5.The step-length each of which can seriously affect the performance.

Besides the size of the response error and the nature of the experimental factors, these are generally not under the control of the experimenter. Often the value of one factor constraints the other .i.e. .the value of the step-length depends on the direction vector. It is therefore necessary to look at how these factors collectively affect performance and the best way each should be determined some of the more recent line search technique that are capable of locating the local optimizer of response surface methodology problem in just one move has been reported by Onukogu (1996) and Onukogu and Esele (1987).

The line search equation

$$x_{i+1} = \bar{x} - \rho_i d_i$$

Where $\underline{x}_i = (x_{i1}, x_{i2}, \ldots, x_{in})'$ is the starting point of the sequence, $d_i = (d_{i1} , d_{i2}, \ldots, d_{in})$ is the direction of search and ρ_i is the step-length.

The line search equation has over the years taken variety of form depending on the direction vector,

$$d_i = T_i g_i$$

Where Ti is a matrix of transformation, gi is the gradient vector at the jth step, when T = I an $n \times n$ identity matrix. The algorithm used in this study makes use of the three basic iterative steps of a line search procedure

(a) To determine the direction vector $\underline{d}_i$ and the step-length, ρ_i at $\underline{x} \in \hat{X}$, $\hat{X}$ is the space of all experimental trials. Perform an experiment to determine the direction of search, d_i.

Move to the point $x_{i+1} = \bar{x} - \rho_i d_i$, ρ_i is the step-length.

(b) If $\underline{x}_1 = x^*$, the local optimizer, stop, otherwise set 1 = 0 and return to step (a) above.

There exist line search algorithms which are capable of reaching the local optimizer of regular unconstrained surface in one move .One of such algorithms is the Minimum Variance Line Search Algorithm (MVA) Onukogu (1997). For constrained optimization problems researchers often apply search techniques that are quite different from experimental design techniques. However, this study evolved a line search algorithm for solving linear programming problems (LPP) a constrained optimization problem which makes use of the principle of optimal experimental design. The new algorithm is known as Minimum Exchange Algorithm (MEA).

THE MINIMUM EXCHANGE ALGORITHM

The aim of this search is to locate the optimum of the response surface with the fewest number of iterations and minimum amount of computation iteration. At first, some ideas are generated concerning which factor or variable that is likely to be important in response surface study. It is usually called the screening experiment. The objective of the factor screening is to reduce the list of variables to a relative few so that the subsequent experiments will be more efficient and require fewer tests. This is identification of the important independent variable. Secondly, the experimental objective is to determine if the current setting of the independent variables results in a value of the response that is near the optimum. If the current settings or levels of the independent variables are not consistent with optimum performance, the experimenter must determine a set of adjustment to the process towards the optimum.

LINEAR PROGRAMMING PROBLEM

Linear programming problem is an optimization problem characterized by linear objective function in non negative variables constrained by a set of m linearly independent equations. Inyama and Osuagwu (1999) states that for every mathematical programming problem the objective function F(x) and constraint g_i (x) , i =1, 2, . . . ,n are always linear.

A linear programming problem is defined as

$$\begin{aligned} &Max(Mini)Z = c_i x_1 + c_2 x_2 + \cdots + c_n x_n. \\ &\quad subject\ to \quad a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq = \geq b_1 \\ &a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq = \geq b_2 \\ &\vdots \quad \vdots \quad \ldots \quad \vdots \quad \ldots \quad \vdots \quad \ldots \quad (5) \\ &a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq = \geq b_m \end{aligned}$$

where

$a_{ij}, b_j, c_j, (i = 1,2,\cdots,m \quad j = 1,2,\cdots,n)$

are constants.
The matrix function

Max (or Min) z = $\underline{C^1 X}$…(6)

Subject to AX (≤ = ≥) b.

Where

$\underline{C}$ is a column vector $\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$

x is a column $\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$\underline{b}$ is a column vector $\begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$

A is mxn scalar matrix $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix}$

STATEMENT OF THE PROBLEM

There are several ways to reach the local optimizer of a response surface by making use of the gradient vector, but a good number of them are uncertain to converge. The problem is to get a better alternative optimal that could be technically or economically more feasible than the other. The first problem is to seek the various ways of locating the optimizers of the response surface. The objective of the study is to seek a more convenient, cost effective and minimum way of locating the optimizer of a response surface. This the study wants to achieve, by using a line search algorithm in solving a numerically hypothetic linear programming problem in comparison with the simplex method approach and also determine the possibility of its convergence at the fewest moves.

SCOPE OF THE STUDY

Within the time frame of this study, and the objective stated above the research trends to limit its scope at locating the optimizer of the response surface with the least number of iteration of the chosen problem at a given minimum amount of computation.

RESEARCH QUESTIONS

1.How can we get a method that will be more convenient and of less computation in locating the optimizer?
2. Can this algorithm be applied on more complicated problems?
3. When compared with other algorithms, can you say that this algorithm is more convenient?
4. Does the algorithm have a convergence point?

LIMITATION OF STUDY

Line search algorithm was used in solving a linear programming problem in this study. Although the study was a success, but it was not without some challenges. Difficulties were encountered in choosing the first point to move, at getting the local optimizer. Also, given that it is relatively new approach in solving linear problems in the world of computing, there was the problem of accessing good literature.

DEFINITION OF TERMS

1. FEASIBLE SOLUTION: All the solution of a linear programming problem that satisfy all the constraints.
2. FEASIBLE REGION: This is the set of all feasible solution.
3. OPTIMAL EXPERIMENTAL DESIGN: It is an essential aspect of an experimental design which enables us to decide whether one design is better than the other.
4. DESIGN MATRIX: This is the values of the initial constrain equation.

MODELS AND THEORIES

When interest of the experimenter is to locate the optimum of the response function. RSM comes in play. The local minimize of a response surface is a point $\underline{x}^* \in \hat{X}$., where $\hat{X}$ is a member of the triplet. i.e.

$$F(\underline{x}^*) = \min f(\underline{x}),\ \underline{x} \in \hat{X}.$$

A fundamental procedure to obtain x* is by the line search equation. Suppose we denote the response by Y which is believed to depend on n independent variables or factors $x_1, x_2, \ldots, x_n$ which span a factor space $s_{(x)}$ and assume that these independent variables are controlled by the experimenter with negligible error. We also assume that the response Y is a random variable with additive error. Then we write

$$Y(X) = f(x_1, x_2, \ldots, x_n\ \theta) + e \ldots(7)$$

Where e is the random error and $f(x_1, x_2, \ldots, x_n)$ is called the response surface, θ is unknown parameter. The problem is to find the local optimizer, $\underline{x}^*$ i.e. levels of the factors $\underline{x}^* = (x^*_1, x^*_2, \ldots, x^*_n)1$ such that for every $\underline{x}$

$$F(\underline{x}^*) \begin{cases} \geq f(\underline{x}) \text{ if } f(x) \text{ is to be maximized} \\ \leq f(\underline{x}) \text{ if } f(x) \text{ is to be minimized} \ldots(8) \end{cases}$$

Onukogu (1997) states the local optimizer

$$\underline{X}^* \in \hat{X}, \quad \ldots(9)$$

where $\underline{x}^*$is the local optimizer, ∈ is the space of the possible trial.

$$F(\underline{x});\ x \in \hat{X}$$

Some of the questions relating to $F(\underline{x})$ that may arise in the course of designing an experiment include,

1.To estimate the value of θ or any subset or linear combination of θ.
2.To determine $\underline{x}^*$ for which $f(\underline{x})$ assumes its optimum value.
No matter the value of the response of an experiment it is desirable that the experiment be designed optimally. That is, to say, the experiment provides maximum information about functions of the parameter of the model.

LITERATURE REVIEW

The use of experimental design method to solve unconstrained optimization problem is so well established that the topic which is usually called Response surface methodology (RSM) has become a permanent feature in every important text on design of experiment. Onukogu (1997), states that response surface methodology can be seen as a bridge linking the subject of experimental design with the subject of unconstrained optimization. In a recent research by Umoren, it is made clear that it’s easy to extend this bridge to a constrained optimization as well. That response surface methodology can be used in solving constrained and unconstrained problem? The field of the response methodology consist of the experimental strategy for exploring the space of the process or independent variables ,empirical statistical modeling to develop an appropriate approximating relationship between the yield and the process variable , an optimization method for finding values process variable that produces desirable values of the response. The interest of the research work is among other things to; locate the optimizer of a linear programming problem - a first order response model. Response surface methodology also deals with exploration of the surface around the optimum point. Montgomery (1976) states that the analysis of response surface can be thought as 'First' climbing a hill' where the top of the hill represent the point of the maximum response, then we may think of descending into a valley. The climbing procedure of the response surface methodology guarantees convergence to a local optimum only. Since he said that convergence is often limited to a demonstration of the steepest ascent (descent) algorithm and, therefore, not comprehensive enough to meet the needs of a student whose interest is more on analytical results. Onukogu (1997) derived from analytical properties, one of such algorithm which is capable of reaching the local optimizer of regular unconstrained surface in one move.

RESEARCHERS IN COMMON

Onukogu(1997), employed the use of Minimum Variance exchange Algorithm in solving unconstrained line search problems. While Umoren (1999) engaged the use of Maximum Norm Exchange Algorithm for solving constrained problems. Also Umoren (1996) used two methods Linear Exchange Line Search Algorithm and The Quadratic Exchange Line Search Algorithm in solving linear programming problem. Tatsuyuki Amajo (2000) carried a research report on sizing optimization using response surface method on first order analysis. Presenting a sizing optimization method based on using first order analysis tools, design engineers can create good design candidate in the concept design stage, while analyzing physical properties of them simultaneously. That the order analysis offers optimal calculation techniques that encourage the use of sizing optimization. In first order analysis the use of an optimization techniques that based on response surface methodology that offer speed convenience as the size optimization.

Response surface methodology in numerical Analysis

The response surface is a type of optimization that applies an approximation technique to the objective and other function of an optimization problem. In order to approximate, it uses a function called response surface. The works of Box & Wilson (1951), actually brought to fore the use of this technique in attaining convergence by repeating numerical and sensitive analysis until the optimal solution is obtained. A common measure of the information contained in any design is in the information matrix of the design defined by

$$M(\xi_n) = \int_{s(x)} \underline{xx}^1 \lambda(x)\, \xi_n\, dx$$

For continuous (approximate) design and

$$M(\xi_n) = \sum_{\underline{x} \in s(\underline{x})} \underline{xx}^1\, \xi_n\, \delta^2_e, \quad M \in M^{nxn}$$

For discrete (exact) design , where $s(x) \in \hat{\mathcal{X}}$ is the experimental area, ξn is a design measure , $\lambda(x)$ is assume to be known bounded positive real-valued continuous function on $\hat{\mathcal{X}}$ and is commonly called the efficiency function. We note that the efficiency function reflects the heteroscedasticity structure in the model, but if $\lambda(x)$ is constant on $\hat{\mathcal{X}}$, we may assume without loss in generality that $\lambda(x) = 1$ and homoscedastic model (Wong, 1992). The continuous function $\lambda(x)$ is equivalent to δ^2_e for exact designs. We note that both the theory and in the construction of the optimum designs, it is convenient to replace the n trials design represented by the design matrix X by a measure over the design region $\hat{\mathcal{X}}$ for n trial design, that is, one for which the weights at the experimental points are integer multiples of 1/n, the measure is denoted by ξn. If the integer restriction is removed the design is denoted by ξ and is referred to as an approximate design since it may not be exactly realizable in practice (Atkinson, 1982). Thus an experimental design is a probability measure ξn on a δ field of set $\hat{\mathcal{X}}$, which include the one point set (Studden and Dette, 1993).

Associated with the design problem is the triplet ($\hat{\mathcal{X}}$, F_x , $\sum_x$) together with the convex optimality criterion function Φ which is selected to reflect the interest of the experimenter. The problem confronting the experimenter is how to select a design so that Φ is minimized. Designs which minimize Φ are called Φ- optimal design.

The definition of the triplet,

$\hat{\mathcal{X}}$ = ($\underline{x}$) is the space of all possible trials, for an n- factor experiment $\underline{x}$ is an n-component vector. Generally, some of the factors can be quantitative or qualitative but here we shall only consider quantitative factors

$F_{\underline{x}}$ = [f(x)] is a space of finite dimensional continuous functions that can be defined in $\hat{\mathcal{X}}$.

$\sum_x = (\delta_x^2)$ is a space of non-negative, continuous random observation error which can be defined in $\hat{\mathcal{X}}$. General, the values of δ^2_x vary from one x to another, but for the purpose of this work δ^2_x shall be assumed to be constant for all x.

(Wong, 1992) Indeed, an optimal design maximizes or minimized an appropriate function of the information matrix or its inverse (Studden and Dette, 1993). Usually, the direct search for an optimal design is difficult, hence it is always necessary for the experimenter to formulate the purpose of the experiment clearly and also choose an appropriate optimality criterion from the class of optimality criteria. One design criterion which has been much studied is that of D-optimality in which the determinant $\det(M(\xi))$ is maximized. Another optimality criterion is the G-optimality which is applicable when the experimenter's interest is on minimizing the maximum variance of the predicted response over the design region. If interest is on minimizing the sum of the variance of the parameter estimates, we speak of A- optimality, where as when interest is on maximizing the minimum Eigen value of $M(\xi)$, we speak of E-optimality. In this study we are making use of the Kiefer(1974) or Atkinson(1988) type Φ optimality criterion which are formulated in terms of the information matrix, the objective being to minimize some functional of $M^{-1}(\xi)$ over ξn. A more detailed account of optimal experimental design can be found in some text books e.g. Pazman (1986), Atkinson and Donev (1992) and Onukogu(1997).

METHODOLOGY

Introduction

The problem is to locate the local optimizer, i.e.

Minimize $x^* \in \hat{\mathcal{X}}$

$$\text{Minimize } f(x) = \underline{c}'\underline{x}$$
$$\text{Subject to } \underline{Ax} = b, \quad \underline{x} \geq 0$$

we seek to locate a minimizer $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ so that $f(x^*) \leq f(x)$, $x \in \hat{\mathcal{X}}$ is the feasible region.

Whatever form the constraints may take, a move is made from the center of a given initial design matrix X along a constant gradient direction and after a finite sequence of moves the algorithm leads to the optimizer of the LP problem. It is shown in this work that the line search techniques which are useful in optimal experimental design can be modified and adapted in solving linear programming problem effectively. Thus to generate a class of line search algorithm that are capable of solving linear programming problem effectively and study the convergence properties of such technique provide the main motivation and objective for this research project. Two commonly used line search algorithms for solving LP problems are the simplex method and Active set method. For some LP problems the simplex method and the active set method will require the use of artificial variables and artificial constraints, in such cases there will be computational demand in solving problems. In addition, for inequality constraints, the simplex method requires the use of slack variables, thus requiring greater computer storage capacity.

Furthermore, the simplex method is potentially an exponential time algorithm and simple examples exist that have some difficulty. It is therefore desirable to develop a new approach for solving LP problem that avoid the above difficulties. We implies a new algorithm [Minimum Exchange Algorithm], the exchange is made on the basis to get the minimum of the support points which form the initial design matrix; that is the support points which has the minimum exchange with the end point of the kth iteration. In development of this new algorithm. We have to exploit some of the similarities between Response Surface Methodology RSM and linear programming problem LPP. These include

(1) Although RSM is usually classified as an unconstrained optimization problem, but in fact the independent variables can only take on values within a finite dimensional region which constraint the acceptable values of the response function. Thus the experimental region in RSM is indeed constrained and plays the same analytical role as the feasible region on LP problem.

(2) Even-though the independent variable in RSM can take on any value and those of linear programming problem can only assume non-negative values both are usually considered to be quantitative, thus

(3) The experimental region in RSM (or equivalent the feasible region linear programming) is a continuous, compact and a metric space. One particular interesting feature of the new algorithm is that all the constraints ($\leq\ =\ \geq$) are treated as equality constraints, i.e. (= = =). thus there is no need for adding slack variables as in some other methods of solving LP problems, like simplex method. Whatever form the constraints may take, a move Is made from the center of a given initial design matrix X along a constraint gradient direction and after a finite sequence of moves the algorithm leads to the optimizer of the linear programming problem.

Minimum Exchange Algorithm

Some basic design principles underlying the development of the algorithm are:

Firstly we do not require more than n, ($n + 1 < n \leq 1/2n\ (n + 1) + 1$) support points for the initial design matrix needed for the commencement of the algorithmic operation; n is the number of variates in the gradient vector of the objective function. This is sustained by Pazman (1986) for equivalent designs.

Secondly, Pazman (1986) has also shown that a design matrix with support points taken at the boundary of the experimental region is better than another design matrix with support points taken at the interior of the experimental region.
The common structure in this algorithm is that it deletes a point $\underline{X}_{mk}$ which is such that

$\|\underline{X}_{mk}\| \geq \|\underline{X}_{ik}\|$, i = 1,2, . . .,n , $i \neq m$

From the initial design matrix X, k = 1, 2, ... And adds to it the point $\underline{X}_K$, the end point of the kth iteration; $\|\underline{X}_{mk}\| \geq \|\underline{X}_{k}\|$
The sequences of steps involved in minimum exchange algorithm are as follows.

S_1 ; at the boundary of $\hat{X}$ centered at $\underline{x}_k = (x_{1k}, x_{2k}, \ldots, x_{nk})$ take n support for the design matrix i.e.

$$x_k = \begin{pmatrix} x_{1k} \\ x_{2k} \\ \vdots \\ x_{mk} \\ \vdots \\ x_{nk} \end{pmatrix}$$

Such that $\det(x'x) \neq 0,\ n+1 < n \leq \frac{1}{2}n(n+1)+1$.

S_2: $x^* = \overline{x_k} - \rho d$

$\bar{X}$ = x/n, k = 1, 2 ,. . .
$\underline{d} = (X'X)^{-1} X^1Y = b$

$$\rho = min\left(\frac{c_i^1 \bar{x}}{c_i^1 d_i} - b_i\right)$$

s_3, $x_k = x^*$ the minimize, stop , otherwise replace x_{mk} in X with x* to have

$$X_{k+1} = \begin{bmatrix} x_{1k} \\ x_{2k} \\ \vdots \\ x^* \\ \vdots \\ x_n \end{bmatrix}$$

$\underline{x}_{k+1} = x_{k=1}/n$,where x_{mk} is such that $\|x_{mk}\| \geq \|x_{ik}\|$, i = 1, 2, . . . , n, $i \neq m$.
S_4: set k + 1 = k and return to step s_2.

The sequence terminates when ever any of the following conditions hold;

(i) $\left\|\frac{x_{+1} - x}{\|x\|}\right\| < \epsilon\ ,\ \epsilon > 0.$
(ii) $|\underline{x}^1_k M^{-1} \underline{x}_k - \underline{x}^1 M^{-1} \underline{x}| < \delta$, $\delta < 0$
(iii) $|x^1_1 M^{-1} x_1 - x M^{-1} X| < \Upsilon$, $\Upsilon > 0$.

We note that the forward procedure of the algorithm is also applicable.

In that case the sequence deletes from the design matrix X the point x_{mk} which is such that

$$\|x_{mk}\| \leq \|x_{ik}\|\ ;\ i = 1,2 \ldots, n, i \neq m$$

And adds to it the point x*, the end point of the k_{th} iteration. This procedure is called the Minimum Exchange Algorithm since the exchange is made with x_{mk} , a point of minimum, i.e.

$\|x^*\| \le \|x\|$

The numerical demonstration reveals the following about the working of the algorithm.

- The determinant of the information matrix decreases from iteration to iteration.
- Each move takes the sequence from point X_k of relatively high d-function to a point of lower d- function.
- At every iteration move is made from the centre of the design to the boundary of the feasible region

The convergence of any sequence must show the following;

1. Must show that the d-function at $(k+1)$ iteration is greater than the d-function at the kth iteration. i.e. $X^1_{k+1} M_k^{-1} X_{k+1} \ge X^1_k X_k^{-1} X_k$.

2.That the determinant of the kth iteration will be greater than the determinant of (k+1) iteration. $det(M_{k+1}) \le det(M_k)$.

Analysis and Results

Demonstration of the procedure using a numerical example.

Example: 4.1

$$\begin{aligned} minimize\ \ f(x) &= 3x_1 + 2x_2 \\ subject\ to\ \ \ 2x_1 + x_2 &\ge 6 \\ x_1 + x_2 &\ge 4 \\ x_1 + 2x_2 &\ge 6 \\ x_1, x_2 &\ge 0 \end{aligned}$$

From the example above, the support points which constitute the initial design matrix are formed from the constrained equation, the points that satisfy the Constrained equations. Let X be the design matrix

$$x = \begin{bmatrix} 3 & 0 \\ 2 & 2 \\ 3 & 3 \\ 0 & 4 \end{bmatrix} \qquad b = \begin{bmatrix} 6 \\ 4 \\ 6 \end{bmatrix}$$

$$\overline{x_k} = (2, 2.25)$$

From our design matrix x_k we get our information matrix

$(M^{-1}\ Or\ x^1 x)$

$$(X_k^1 X_k) = \begin{bmatrix} 3 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 2 & 2 \\ 3 & 3 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 22 & 13 \\ 13 & 29 \end{bmatrix}$$

We normalize it by dividing $(x^1 x)/2$

$$\frac{(x_k^1 x_k)}{2} = \begin{bmatrix} 22/2 & 13/2 \\ 13/2 & 29/2 \end{bmatrix}$$

To get the b estimate or d_i

$$y = \begin{bmatrix} 9 \\ 10 \\ 15 \\ 8 \end{bmatrix}$$

$$b = (x_k^1 x_k)^{-1} x^1 y$$

$$|x_k^1 x_k| = \begin{bmatrix} 29/2 & -13/2 \\ -13/2 & 22/2 \end{bmatrix}$$

$$\det(x_k^1 x_k) = (29/2 \times 22/2) - (-13/2 \times -13/2) = 117.25$$

$$(x_k^1 x_k)^{-1} = \begin{bmatrix} 0.1237 & -0.0554 \\ -0.0554 & 0.0938 \end{bmatrix}$$

$$b \sim d, \qquad b = (x_k^1\ x_k)^{-1} x_k^1 y$$

$$x_k^1 y = \begin{bmatrix} 3 & 2 & 3 & 0 \\ 0 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 9 \\ 10 \\ 15 \\ 8 \end{bmatrix} = \begin{bmatrix} 92 \\ 97 \end{bmatrix}$$

$$b = \begin{bmatrix} 0.1237 & -0.0554 \\ -0.0554 & 0.0938 \end{bmatrix} \begin{bmatrix} 92 \\ 97 \end{bmatrix} = \begin{bmatrix} 6.0066 \\ 4.0018 \end{bmatrix}$$

$$b = \begin{bmatrix} 6.0066 \\ 4.0018 \end{bmatrix}$$

$$d_1 = \frac{b_1}{\sqrt{b_1^2 + b_2^2}} = \frac{6.0066}{\sqrt{6.0066^2 + 4.0018^2}} = 0.8322$$

$$d_2 = \frac{b_2}{\sqrt{b_1^2 + b_2^2}} = \frac{4.0018}{\sqrt{6.0066^2 + 4.0018^2}} = 0.5532$$

$$d = \begin{bmatrix} 0.83 \\ 0.55 \end{bmatrix}$$

Using the line search equation,

$$x^* = \bar{x} - \rho_i d_i$$

$$\rho_1 = \frac{(2 \quad 1)\begin{bmatrix} 2.00 \\ 2.25 \end{bmatrix} - 6}{(2 \quad 1)\begin{bmatrix} 0.83 \\ 0.55 \end{bmatrix}} = 0.1126$$

$$\rho = min\left(\frac{c_i^1 \bar{x}}{c_i^1 d_i} - b_i\right)$$

$$\rho_2 = \frac{(1 \quad 1)\begin{bmatrix} 2.00 \\ 2.25 \end{bmatrix} - 4}{(1 \quad 1)\begin{bmatrix} 0.83 \\ 0.55 \end{bmatrix}} = 0.1812$$

$$\rho_3 = \frac{(1 \quad 2)\begin{bmatrix}2.00\\2.25\end{bmatrix}}{(1 \quad 2)\begin{bmatrix}0.83\\0.55\end{bmatrix}} - 6 = 0.2591$$

A move is now made to

$$X^* = \overline{X_1} - \rho_1 d_1$$
$$\overline{x_1} = \begin{bmatrix}2.00\\2.25\end{bmatrix}$$

Using a line search equation, we substitute the values

$$x_1^* = \overline{X_1} - \rho_1 d_1$$
$$x_1^* = \begin{bmatrix}2.00\\2.25\end{bmatrix} - (0.1126)\begin{bmatrix}0.83\\0.55\end{bmatrix} = \begin{bmatrix}1.91\\2.19\end{bmatrix}$$

Subjecting the values of x^*_1 into the objective function.

$F(x^*_1, x^*_2) = f(1.91, 2.19) = 10.10$

We exchange the point (3.00 3.00) with (1.91 2.19) to get a new design

$$X_{k+1} = \begin{bmatrix}3.00 & 0.00\\1.91 & 2.19\\2.00 & 2.00\\0.00 & 4.00\end{bmatrix}$$

$$(x^1_{k+1}x_{k+1}) = \begin{bmatrix}3.00 & 1.91 & 2.00 & 0.00\\0.00 & 2.19 & 2.00 & 4.00\end{bmatrix}\begin{bmatrix}3.00 & 0.00\\1.91 & 2.19\\2.00 & 2.00\\0.00 & 4.00\end{bmatrix} = \begin{bmatrix}16.6481 & 8.1829\\8.1829 & 24.7961\end{bmatrix}$$

$$(x^1_{k+1}x_{k+1})/2 = \begin{bmatrix}16.6481/2 & 8.1829/2\\8.1829/2 & 24.791/2\end{bmatrix}$$

$$|x^1_{k+1}x_{k+1}| = (12.398 \times 8.3242) - (-4.0915 \times -4.0915) = 86.46$$

$$\det(x^1_{k+1}x_{k+1}) = 86.46$$
$$(x^1_{k+1}x_{k+1})^{-1} = \begin{bmatrix}0.1434 & -0.0473\\-0.0473 & 0.0963\end{bmatrix}$$

Starting point is x_{k+1} = (1.73 2.05) and the step is calculated as before to be $\rho_2 = 0.1812$

$$x_2^* = \begin{bmatrix}1.73\\2.05\end{bmatrix} - (0.1812)\begin{bmatrix}0.83\\0.55\end{bmatrix} = \begin{bmatrix}1.58\\1.95\end{bmatrix}$$

Subjecting the values of x^*_2 into the objective function.

$$f(x_1^* \quad x_2^*) = f(1.58 \quad 1.95) = 8.64$$

We exchange the point (2.00 2.00) with (1.58 1.95) to get a new design matrix.

$$X_{k+2} = \begin{bmatrix}3.00 & 0.00\\1.91 & 2.19\\1.58 & 1.95\\0.00 & 4.00\end{bmatrix}$$

$$(x^1_{k+1}x_{k+1}) = \begin{bmatrix}3.00 & 1.91 & 1.58 & 0.00\\0.00 & 2.19 & 1.95 & 4.00\end{bmatrix}\begin{vmatrix}3.00 & 0.00\\1.91 & 2.19\\1.58 & 1.95\\0.00 & 4.00\end{vmatrix} = \begin{bmatrix}15.1445 & 7.2639\\7.2639 & 24.5986\end{bmatrix}$$

$$(x^1_{k+2}x_{k+2})/2 =$$
$$\begin{bmatrix}15.1445/2 & 8.1829/2\\7.2639/2 & 24.5986/2\end{bmatrix} = \begin{bmatrix}7.5723 & 3.6320\\3.6320 & 12.2993\end{bmatrix}$$
$$\det(x^1_{k+2}x_{k+2}) - (7.5723 \times 12.2993 - 7.5723 \times 7.5723) - 79.94$$

$$M^{-1} = \begin{bmatrix}0.1539 & -0.0454\\-0.0454 & 0.0947\end{bmatrix}$$

The step lenght of $\overline{x_{k+2}}$ = (1.62 2.04) *the calculated* $\rho_3 = 0.2591$

$$x_3^* = \begin{bmatrix}1.62\\2.04\end{bmatrix} \quad (0.2591)\begin{bmatrix}0.83\\0.55\end{bmatrix} = \begin{bmatrix}1.40\\1.90\end{bmatrix}$$

Subjecting the value of X_3^* into the objective function,

$$f(x_1^* \quad x_2^*) - f(1.40 \quad 1.90) - 7.91$$
$$x_1 = x^* \text{ the minimizer is } \begin{bmatrix}1.40\\1.90\end{bmatrix} = 7.91$$

Table 4.1: The determinant of the information matrix at the kth iteration in the Minimum Exchange Algorithm

iteration (k)	det(M_k)
1	117.25
2	86.46
3	79.94

Table 4.2: The d- function at the end point of the kth iteration of Minimum Exchange Algorithm

Iteration k	end point x_k		$x^1_k M^{-1}k\, x_k$
1	1.91	2.19	0.44
2	1.58	1.95	0.43
3	1.40	1.90	0.40

Comparison of performance of the minimum exchage algorithm and the simplex method

Solving Example 1.0 with simplex method, we got the following values for;

$$\min z = 3x_1 + 2x_2$$
$$\text{subject to} \quad 2x_1 + x_1 \geq 6$$
$$x_1 + x_2 \geq 4$$
$$x_1 + 2x_2 \geq 6$$
$$x_1\ x_2 \geq 0$$

Put it in standard form.

$$\min z = 3x_1 - 2x_2 + 0s_1 + 0s_2 + 0s_3$$
$$\text{subject to} \quad 2x_1 + x_1 + s_1 = 6$$
$$x_1 + x_2 + s_2 = 4$$
$$x_1 + 2x_2 + s_3 = 6$$
$$x_1\ x_2\ s_1\ s_2\ s_3 \geq 0$$

	Non- basic			basic			
basic	Z	X_1	X_2	S_1	S_2	S_3	b
Z	1	-3	-2	0	0	0	RHS
S_1	0	(2)	1	1	0	0	6
S_2	0	1	1	0	1	0	4
S_3	0	1	2	0	0	1	6
Z	1	0	-1/2	3/2	0	0	9
X_1	0	1	½	½	0	0	3
S_2	0	0	3/2	0	-1	0	7
S_3	0	0	(5/2)	½	0	1	(9)
Z	1	0	0	7/5	0	-1/5	54/5
X_1	0	1	0	3/5	0	1/5	6/5
S_2	0	0	0	-3/5	1	3/5	8/5
X_2	0	0	1	1/5	0	2/5	18/5

The optimal solution is X_1 = 6/5 = 1.2, x_2 = 18/5 = 3.6, When subjected to the objective function we have f (1.2 3.6) Min f(x)= 10.80

Table 4.4: Comparison of the result of Minimum Exchange Algorithm Performance and the simplex method

Algorithm	Value of minimize	Value of f(x)
MEA	1.40 I.90	7.99
Simplex method	1.20 3.60	10.80

From the result, the minimizer of the Simplex method is 10.80, while that of the minimum exchange algorithm is 7.91. Therefore the minimum exchange algorithm is more efficient in terms of its minimum value compared with the value of the simplex method.

Summary

The problem of developing a class of exchange algorithm for solving linear problem using an experimental design procedure has been the main focus of this research project. The totality of the report may be summarized as follows:

The new algorithm: Minimum Exchange Algorithm. In developing this algorithm the similarities between response surface methodology and linear programming problem were exploited. Some similarities include the fact that both the experimental region in response surface methodology and the feasible region in linear programming play the same analytical role. Both the objective function in linear programming and response function in response surface methodology problem are continuous non-stochastic functions defined in a finite dimensional space.

We notice the following about the algorithmic operations of the algorithms referred to;

(a) The sequence moves from a point $\underline{X}_k$ of relatively high variation to a point $\underline{X}_{k+1}$ of lower variation. i.e. $x_{k+1}^1 M_k^{-1} x_k \geq x_k^1 M_k^{-1} x_k$

Where M^{-1}_k and $\underline{X}_k$, are the information matrix and the end point of the iteration, respectively.

(b) The determinant of the information matrix at the kth iteration, $\det(M_K)$ is non-increasing function.

(c) That the decrease in the $detM^{-1}$ produces a corresponding increase in d-function $\underline{X}_k^1$ M^{-1} $\underline{X}_k$; i.e. the sequence $(X_k^1 M^{-1} X_k)_{K=1 to \infty}$ is non – decreasing.

(d) The sequence $(\underline{X}_k)_{k=1}^{\infty}$ is bounded and has a limiting value X_c which has been shown to be not different from $\underline{X}^*$ the minimize of F($\underline{x}$).

Some optimality conditions to be satisfied by the minimize of a linear programming problem has be obtained: one of such conditions is that, If $\underline{X^*}$ is a minimizer of the linear programming problem, then the d-function of the objective function at $\underline{X}^*$ is less than the d-function at any other point within the feasible region. $\underline{X}^{*1} M^{-1} \underline{X}^* < \underline{X}^1 M^{-1} \underline{X}$

A numerical comparison of the algorithm (MEA) with the simplex method, revealed that the new algorithm compare favorable than the simplex method.

Conclusion

Using the experimental design procedure this work has evolved a new algorithm namely; Minimum Exchange Algorithm for solving linear programming problem. The algorithm has been shown to converge compared with simplex method for solving linear programming problem. We conclude therefore that using the line search algorithm in solving linear programming problem is more convenient to getting its minimum when compared with other algorithm. Also we discovered that the experimental design procedure is simple and gives a more minimal value than the other, like the simplex method.

Recommendation

This study strongly recommends the use of line search algorithm for solving linear programming problem.

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